

Math 1260 Quiz #12 - June 19, 2006

1. Let $f(x) = \frac{1}{x}$. Show *using the definition of the derivative*, that $f'(x) = \frac{-1}{x^2}$.

Solution:

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{-h}{x(x+h)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} \\ &= \frac{-1}{x^2} \end{aligned}$$

2. Determine the equation of the tangent line to $y = \frac{1}{x}$ at the point where $x = 3$.

The point is $(3, 1/3)$, the slope is $f'(3) = -1/3^2 = -1/9$. Thus:

$$y - 1/3 = -1/9(x - 3)$$