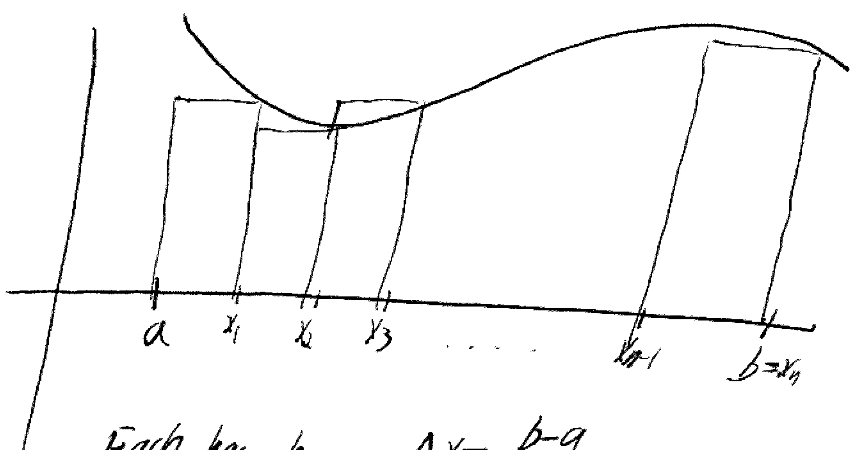
11/21/06

1. Go over exam
2. Final ≈ 1/2 chapter 4, 1/2 cumulative

Area Under Curve

Problem Given $y = f(x) \geq 0$. Find area under $y = f(x)$ from $a \leq x \leq b$

Procedure 1. Divide $[a,b]$ into n ~~rectangles~~ intervals equal length
2. Approximate by areas of rectangles
3. let $n \to \infty$, take limit



Each ~~has~~ base $\Delta x = \frac{b-a}{n}$

$x_1 = a + \Delta x$

$x_2 = a + 2\Delta x$

$x_3 = a + 3\Delta x$

$\vdots$

$x_{n-1} = a + (n-1)\Delta x$

$x_n = a + n\Delta x = b$

Total area $= R_n = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x = \Delta x \sum_{i=1}^{n} f(x_i)$

Def: The area of region S under a continuous function $f(x)$ is

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\Delta x)\,\Delta x$$

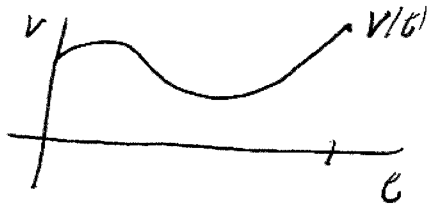
$$\Delta x = \frac{b-a}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + \frac{i(b-a)}{n}\right)\frac{b-a}{n}$$

Theorem: Suppose $f(x)$ is continuous. Then

1. This limit always exists.
2. The limit is the same even if we choose left sides of rectangles or midpoints, or in fact any point $x_i^* \in [x_{i-1}, x_i]$

Recall: If



then area = total distance travelled

Problems

#9

t (hours)	0	2	4	6	8	10
rate (L/h)	8.7	7.6	6.8	6.2	5.7	5.3

Give upper and lower estimates for total amount that leaked

#15 Find a region whose area is

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan\left(\frac{i\pi}{4n}\right)$$

#16 Estimate area under $y = x^2 + x$ from $x = 0$ to 4

using 10 rectangles