

Name:

# SOLUTIONS

Quiz #12 - April 20, 2007

1. Find the radius of convergence and interval of convergence of the power series:

$$\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{4^n \ln n}$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{\frac{x^{n+1}}{4^{n+1} \ln(n+1)}}{\frac{x^n}{4^n \ln n}} \right| = \left| \frac{x \cdot \ln n}{4 \cdot \ln(n+1)} \right| \quad \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{x}{4} \right|$$

Need  $\left| \frac{x}{4} \right| < 1$   $-4 < x < 4$

$R=4$

$x=4$   $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$  CONVS AST

$x=-4$   $\sum_{n=2}^{\infty} \frac{1}{\ln n}$  DIVS

$(-4, 4]$

2. Find a power series representation for  $f(x) = \frac{x^2}{3+x}$  and determine the interval of convergence.

$$\frac{x^2}{3+x} = \frac{x^2}{3} \cdot \frac{1}{1 + \frac{x}{3}} = \frac{x^2}{3} \cdot \frac{1}{1 - (-\frac{x}{3})}$$

$$= \frac{x^2}{3} \cdot \left( 1 - \frac{x}{3} + \frac{x^2}{9} - \frac{x^3}{27} + \frac{x^4}{81} \dots \right)$$

$$= \frac{x^2}{3} - \frac{x^3}{9} + \frac{x^4}{27} \dots$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot x^{n+1}}{3^n}$$

Need  $\left| \frac{x}{3} \right| < 1$   $-3 < x < 3$

$R=3$

$(-3, 3)$

$x=3$   
 $x=3$

both diverge

since an is not even away to 0.