

Name: SOLUTIONS

Quiz #4 - February 1, 2007

1. A bacteria culture grows with constant relative growth rate. After 3 hours there are 500 bacteria and after 6 hours there are 50,000.
- Find a formula for the population of bacteria after t hours.
 - Find the initial population.
 - Find the number of bacteria after 8 hours.
 - At what time will the population reach 300,000.?
 - Find the rate of growth in the number of bacteria at time $t = 5$ hours.

$P(t) = P_0 e^{kt}$. Given that $P(3) = 500$ $P(6) = 50000$.

$$\begin{aligned} 500 &= P_0 e^{3k} \\ 50000 &= P_0 e^{6k} \end{aligned} \rightarrow 100 = e^{3k} \quad k = \frac{1}{3} \ln 100$$

$$P(t) = P_0 e^{\frac{1}{3} \ln 100 t} = P_0 \cdot 100^{t/3}$$

$$500 = P_0 \cdot 100^{3/3} \quad P_0 = 5$$

a. $P(t) = 5 \cdot 100^{t/3}$

b. 5

c. $P(8) = 5 \cdot 100^{8/3} = 2118443$

d. $300000 = 5 \cdot 100^{t/3}$

$$60000 = 100^{t/3} \quad 3 \cdot \frac{\ln 60000}{\ln 100} = t \rightarrow 7.17 \text{ hours}$$

e. $P'(t) = 5 \cdot 100^{t/3} \cdot \frac{1}{3} \ln 100$

$P'(5) = 825142.7$ bact/hour

2. a. Evaluate $\sin^{-1}(1/2)$.

b. What is $\frac{d}{dx} \sin^{-1} x$.

a. $\pi/6$

b. $\frac{1}{\sqrt{1-x^2}}$