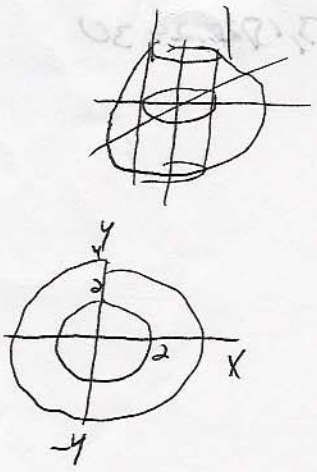
Name: SOLUTIONS

Quiz #8 - November 5, 2004

1. Use polar coordinates to find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 16$ and *outside* the cylinder $x^2 + y^2 = 4$



In xy plane the region is $0 \le \theta \le 2\pi$, $2 \le r \le 4$.

$z = \sqrt{16-r^2}$ is top $z = -\sqrt{16-r^2}$ is bottom.

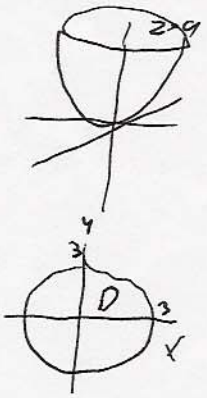
$$\int_0^{2\pi}\int_2^4 \sqrt{16-r^2} - -\sqrt{16-r^2}\; r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_2^4 2r\sqrt{16-r^2}\,dr\,d\theta = \int_0^{2\pi} -\frac{2}{3}(16-r^2)^{3/2}\Big|_2^4\,d\theta$$

$$= \int_0^{2\pi} 0 - -\frac{2}{3}\cdot(12)^{3/2}$$

$$= \boxed{2\pi \cdot \frac{2}{3} \cdot 12^{3/2}}$$

2. Find the surface area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$.



$$S.A. = \iint_D \sqrt{1+4x^2+4y^2}\,dA$$

$$= \int_0^{2\pi}\int_0^3 \sqrt{1+4r^2}\; r\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{2}{3}\cdot\frac{1}{8}(1+4r^2)^{3/2}\Big|_0^3\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{12}\cdot 37^{3/2} - \frac{1}{12} = 2\pi\left(\frac{1}{12}\cdot 37^{3/2} - \frac{1}{12}\right)$$

$$= \frac{\pi}{6}\left(37^{3/2} - 1\right)$$