Name: SOLUTIONS

Quiz #9 - December 3, 2004

1. Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

a. $\mathbf{F}(x,y) = (x\cos(y)), \cos(x))$.

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = -\sin x - (-x\sin y) \neq 0$$

Not conservative

b. $\mathbf{F}(x,y) = (y - \sin(x), x + 1)$. $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1 - 1 = 0$ (conservative)

$$f(x,y) = \int y - \sin x \, dx = yx + \cos x + g(y)$$

$$f(x,y) = \int x + 1 \, dy = yx + y + h(x)$$

$f(x,y) = yx + y + \cos x$

2. Define what a conservative vector field is.

$\vec{F}$ is conservative if $\vec{F} = \nabla f$ for some f.

3. Let $\mathbf{F}(x,y) = (xe^{xy}4y + \cos(x) + \sqrt{x+1})$. Let C be the curve given by $\mathbf{r}(t) = (t, t^3)$ for $1 \leq t \leq 2$. Calculate:

$$\int_c \nabla \mathbf{F} \cdot d\mathbf{r}$$

$$\int_C \nabla F \cdot dr = F(r(2)) - F(r(1)) =$$

$$F(2,8) - F(1,1)$$

$$= 2e^{16} + 8 + \cos(2) + \sqrt{3} - e - 1 - \cos 1 - \sqrt{2}$$