Linear Algebra (Math 2890) Practice Problems 1

1. (a) Solve the following system of equations:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 6 \end{bmatrix}$$

   (b) Explain your answer in (a) in terms of the column vectors of the corresponding matrix.

2. Let $M = \begin{bmatrix} 1 & 3 & -4 & 7 \\ 2 & 6 & 5 & 1 \\ 3 & 9 & 4 & 5 \end{bmatrix}$. Describe all the solutions of $M\vec{x} = 0$

3. Consider a linear system whose augmented matrix is of the form

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & 2 & 0 & 1 \\ 3 & 5 & a & b \end{array}\right]$$

   (a) For what values of a will the system have a unique solution? What is the solution?(your answer may involve a and b)

   (b) For what values of a and b will the system have infinitely many solutions?

   (c) For what values of a and b will the system be inconsistent?

4. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

   Find AB, B^T and B^TA^T.

5. Determine if the columns of the matrix form a linearly independent set. Justify your answer.

$$\begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -4 & -3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & 3 \\ 5 & 4 & 6 \end{bmatrix}, \begin{bmatrix} -4 & -3 & 1 & 5 & 1 \\ 2 & -1 & 4 & -1 & 2 \\ 1 & 2 & 3 & 6 & -3 \\ 5 & 4 & 6 & -3 & 2 \end{bmatrix}.$$

6. Determine if the following matrices are invertible.

$$M = \begin{bmatrix} 0 & 1 & 0 & t \\ -1 & 0 & t & 0 \\ 0 & -t & 0 & 1 \\ -t & 0 & -1 & 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

7. Let $A = \begin{bmatrix} 3 & 7 & -3 \\ 3 & 8 & -1 \\ -6 & -14 & 7 \end{bmatrix}$.

   (a) Compute A^{-1}.

   (b) Is $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ lies in the span of the column vectors of A?

8. $M = \begin{bmatrix} 1 & 1 & 2 \\ 1 & a+1 & 3 \\ 1 & a & a+1 \end{bmatrix}$.

   (a) Describe the values of a so that M is invertible.

   (b) Describe the values of a so that the column vectors of M are linear independent.

9. Let $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Suppose $T : R^3 \mapsto R^2$ is a linear transformation such that $T(e_1 + e_2) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $T(e_1 - e_2) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T(e_1 + e_2 + e_3) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. What is $T(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix})$?