Linear Algebra (Math 2890) Review Problems II

1. (a) Show that the matrix $\begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$ is invertible and find its inverse.

   (b) Use previous result to find the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$.

2. (a) Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 2 & 8 \\ 2 & 6 & 23 \end{bmatrix}$. Find an LU factorization for A.

   (b) Use LU decomposition to find the solution of $Ax = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

3. Find all values of a and b so that the subspace of $\mathbb{R}^4$ spanned by $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} b \\ 1 \\ -a \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \end{bmatrix} \right\}$ is two-dimensional.

4. Let $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \}$. You can assume that $\mathcal{B}$ is a basis for R^3

   (a) Which vector x has the coordinate vector $[x]_B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

   (b) Find the β-coordinate vector of $y = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$

5. Let

$$M = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 1 & 2 & 5 & 1 \\ 1 & 3 & 7 & 2 \end{bmatrix}$$

   Find bases for $Col(M)$ and $Nul(M)$, and then state the dimensions of these subspaces

6. Determine which sets in the following are bases for $\mathbb{R}^2$ or $\mathbb{R}^3$. Justify your answer

(a) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \end{bmatrix}$. (b) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$.

(c) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. (d) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(e) $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.

7. Let A be the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

(a) Find the characteristic equation of A.

(b) Find the eigenvalues and a basis of eigenvectors for A.

8. Let A be the matrix

$$A = \begin{bmatrix} -3 & -4 \\ -4 & 3 \end{bmatrix}.$$

(a) Find the eigenvalues and a basis of eigenvectors for A.

(b) Diagonalize the matrix A if possible.

(c) Find the matrix exponential e^A.

9. Find a good approximation for the vector $\begin{bmatrix} .8 & .6 \\ .2 & .4 \end{bmatrix}^n \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ for n very large (say $n = 100$).