1.2 Congruence of Triangles
August 31, 2012

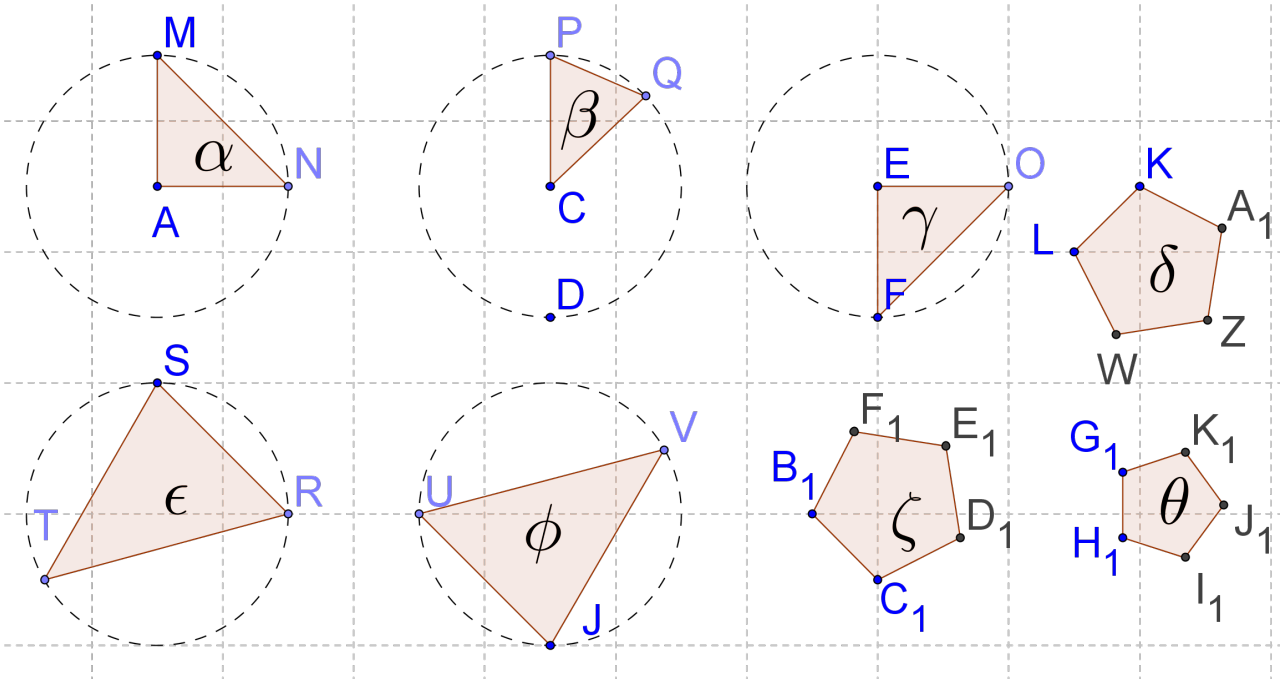

Figure 1:

Example 1 *What is the relationship between the above figures?*

We will start by investigating "congruent triangles".

Definition 1 *Triangles are congruent if there exists a one-to-one correspondence between their vertices so that corresponding sides are congruent and corresponding angles are congruent. If two triangles $\triangle ABC$ and $\triangle DEF$ are congruent, we denote it by $\triangle ABC \cong \triangle DEF$.*

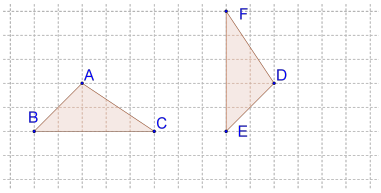

Figure 2:

Example 2 *In the figure above, $\triangle ABC$ and $\triangle DEF$ are congruent. We have $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$. Obviously, we also have $\triangle BCA \cong \triangle EFD$.*

Obviously, two triangles are congruent if their corresponding angles and sides are congruent. But we don't need to verify all these 6 congruent relation to show that two triangles are congruent. It turns out that congruence of fewer corresponding parts is sufficient to determine that two triangles are congruent.

We start with the following axiom.

Axiom 1 ***(The Side, Angle, Side (SAS) Correspondence Condition)*** *If two sides and the angle included between these sides are congruent to two sides and that the included angles of the second triangle, then the triangles are congruent*

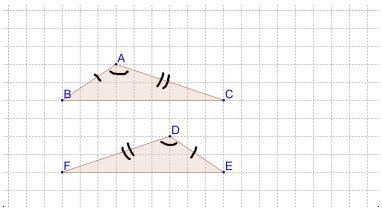

Figure 3:

Before we continue, let us introduce some terms for special triangles.

Definition 2 *1. A triangle is isosceles if at least two of its sides are congruent.*
2. If all the sides of a triangle are congruent, the triangle is equilateral.
3. A triangle with no two sides congruent is called scalene.
4.A triangle with all acute angles is called an acute triangle.
5. A triangle with an obtuse angles is called an obtuse triangle.
6. A triangle with a right angles is called a right triangle.

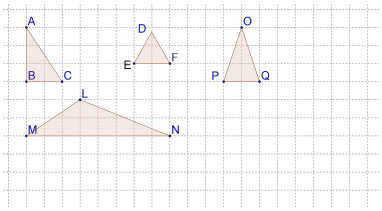

Figure 4:

Example 3 *Describe those triangles above?*

$\triangle ABC$

$\triangle DEF$

$\triangle OPQ$

$\triangle LMN$

Theorem 0.1 ***The Isosceles Triangle Theorem*** *If two sides of a triangle are congruent, then the angles opposite to these sides are congruent.*

Proof. Let $\triangle ABC$ be a triangle that $\overline{AB}$ and $\overline{AC}$ are congruent. We want to that $\angle B = \angle C$. Let consider the triangle $\triangle ACB$. Since $\overline{AB} \cong \overline{AC}$, $\overline{AC} \cong \overline{AB}$ and $\angle BAC = \angle CAB$, we have $\triangle ABC \cong \triangle ACB$ by SAS. This implies that $\angle B = \angle C$. □

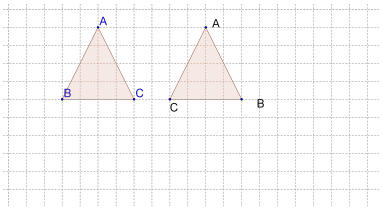

Figure 5: