Review Problems for Midterm I

MATH 2850 – 004

The detail of the information about the first midterm can be found at
http://www.math.utoledo.edu/~mtsui/calc06sp/exam/midterm1.html
The first midterm will cover 13.1, 13.2, 13.3, 13.4 and 13.5.
The first midterm will be held on Feb. 1 (Wednesday) in class.
Online hw13.5 will be due Jan 30 (Monday), 12 p.m.
You should also review the homework problems to prepare for the midterm.

(1) Two forces, represented by the vectors $\overrightarrow{F_1} = 8\overrightarrow{i} - 6\overrightarrow{j}$ and $\overrightarrow{F_2} = 3\overrightarrow{i} + 2\overrightarrow{j}$, are acting on an object. Give a vector representing the force that must be applied to the object if it is to remain stationary.

(2) Let $\overrightarrow{a}$ and $\overrightarrow{b}$ be the vectors $\overrightarrow{a} = <1, 2>$ and $\overrightarrow{b} = <1, -1>$.
  (a) Find numbers r and s such that $\overrightarrow{v} = r\overrightarrow{a} + s\overrightarrow{b}$ if $\overrightarrow{v} = <2, 1>$
  (b) Describe the set of vectors $\{\overrightarrow{w} = s\overrightarrow{u} + t\overrightarrow{v} \mid -2 \le s \le -1, 1 \le t \le 2\}$ geometrically.
  (c) Describe the set of vectors $\{\overrightarrow{w} = 2\overrightarrow{u} + t\overrightarrow{v} \mid 0 \le t \le \frac{2}{3}\}$ geometrically.

(3) Let S be the triangle with vertices $A = (2, 2, 2)$, $B = (4, 2, 1)$ and $C = (2, 3, 1)$.
  (a) Find the length of the shortest side of S.
  (b) Find the cosine of the angle BAC at vertex A.
  (c) Find the area of the triangle ABC.
  (d) Find a vector that is perpendicular to the plane that contains the points A, B and C.
  (e) Let $D = (3, 1, 1)$. Determine whether A, B, C and D lie on the same plane.
  (f) Let $E = (6, 3, -1)$. Determine whether A, B, C and E lie on the same plane.
  (g) Find the volume of the parallelepiped formed by $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$.

(4) If $\overrightarrow{v} \times \overrightarrow{w} = 2\overrightarrow{i} - 3\overrightarrow{j} + 5\overrightarrow{k}$ and $\overrightarrow{v} \cdot \overrightarrow{w}$ = 3, then find $\tan(\theta)$ where θ is the angle between $\overrightarrow{v}$ and $\overrightarrow{w}$.

(5) Show that the vectors $(\overrightarrow{b} \cdot \overrightarrow{c})\overrightarrow{a} - (\overrightarrow{a} \cdot \overrightarrow{c})\overrightarrow{b}$ and $\overrightarrow{c}$ are perpendicular.

(6) A 100-meter dash is run on a track in the direction of the vector $\overrightarrow{v} = 2\overrightarrow{i} + 6\overrightarrow{j}$. The wind velocity $\overrightarrow{w}$ is $5\overrightarrow{i} + \overrightarrow{j}$ km/h. The rules say that a legal wind speed measured in the direction of the dash must not exceed 5 km/h. Will the race results be disqualified due to an illegal wind.

(7) What is known about θ, the angle between two nonzero vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ if
 (a) $\overrightarrow{a} \cdot \overrightarrow{b} = 0$?
 (b) $\overrightarrow{a} \cdot \overrightarrow{b} > 0$?
 (c) $\overrightarrow{a} \cdot \overrightarrow{b} < 0$?
 (d) $\overrightarrow{a} \times \overrightarrow{b} =< 0, 0, 0 >$

(8) (a) Find a vector equation of the line through $(2, 4, 1)$ and $(4, 5, 3)$
 (b) Which of the following pairs of points lies on a line that is parallel to the line through $(2, 4, 1)$ and $(4, 5, 3)$. (a) $(13, 14, 12)$, $(12, 12, 11)$ (b) $(7, 8, 6)$, $(8, 8, 7)$ (c) $(9, 10, 8)$, $(13, 12, 12)$.
 (c) Find a vector equation of the line through $(1, 1, 1)$ that is parallel to the line through $(2, 4, 1)$ and $(4, 5, 3)$.
 (d) Find a vector equation of the line through $(1, 1, 1)$ that is parallel to the line $\frac{x-2}{2} = -\frac{y}{1} = \frac{z-2}{2}$.

(9) (a) Find the equation of a plane that intersects the xz-plane along the line $z = 2x - 3$ and contains the point $(1, 2, 1)$.
 (b) Find the equation of a plane perpendicular to the vector $\overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}$ and passing through the point $(1, 1, 1)$.
 (c) The angle between two intersecting planes is defined to be the (acute) angle made by their normal vectors. Find the angle between the planes $3x + 2y - z = 7$ and $x - 4y + 2z = 0$.
 (d) Find the equation of a plane perpendicular to the planes $3x+2y-z = 7$ and $x - 4y + 2z = 0$ and passing through the point $(1, 1, 1)$.

(10) (a) Find the distance between the point $(1, 2, 3)$ and the plane $2x-2y+z = 7$.
 (b) Find the distance between the planes $2x-y+2z = 10$ and $4x-2y+4z = 7$.
 (c) A plane P is drawn through the points $A = (1, -1, 0)$, $B = (0, 1, -1)$ and $C = (1, 0, -1)$. Find the distance between the plane P and the point $(1, 1, 1)$.

(11) (a) Find the rectangular coordinates of the point with cylindrical coordinates $(4, \frac{pi}{6}, -2)$.

(b) Find spherical coordinates of the point with rectangular coordinates . $(-3, 3, 3\sqrt{6})$.
(c) Write the equation $x^2 + y^2 + z^2 - 2z = 1$ in cylindrical coordinates.
(d) Write the equation $x^2 + y^2 + z^2 - 2z = 1$ in spherical coordinates.

(12) Determine whether the sphere $(x-2)^2+(y-1)^2+(z+1)^2 = 4$ intersects with the following planes

$$P_1 : 2x + 2y - z = 1,$$
$$P_2 : 2x + 2y - z = -1,$$

and

$$P_3 : 2x - y + 2z = 4.$$

Hint: First compute the distance between the center of the sphere and the plane. Then compare the distance and the radius of the sphere.