Review Problems for Midterm II
MATH 2850 – 004

The detail of the information about the second midterm can be found at http://www.math.utoledo.edu/~mtsui/calc06sp/exam/midterm2.html

You should also review the homework and quiz problems to prepare for the midterm.

This midterm will cover 13.7 , Chapter 14 and 15.1-15.7.

(1) (a) Change each of the following points from rectangular coordinates to cylindrical coordinates and spherical coordinates:

$(2, -1, 2), (2, -2, -3)$.

(b) Convert the equation $\cos(\phi) = \sin(\theta)$ into rectangular coordinates.

(c) Convert the equation $r \cos(\theta) = z$ into rectangular coordinates.

(2) Find the arc-length of the curve $r(t) = \langle 2t, e^t, e^{-t} \rangle$ when $0 \leq t \leq \ln(2)$.

(3) (a) Find parametric equations for the tangent line to the curve $r(t) = \langle t^3, t, t^3 \rangle$ at the point $(-1, 1, -1)$.

(b) At what point on the curve $r(t) = \langle t^3, t, t^3 \rangle$ is the normal plane (this is the plane that is perpendicular to the tangent line) parallel to the plane $24x + 2y + 24z = 3$?

(4) Find the unit tangent, unit normal, binormal vectors and curvature of the curve $r(t) = \langle 4t, \cos(3t), \sin(3t) \rangle$.

(5) Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(1, 2, 2)$ and use it to estimate $\sqrt{(1.1)^2 + (2.1)^2 + (1.9)^2}$.

(6) (a) Find the equation for the plane tangent to the surface $z = 3x^2 - y^2 + 2x$ at $(1, -2, 1)$.

(b) Find the equation for the plane tangent to the surface $x^2 + xy^2 + xyz = 4$ at $(1, 1, 2)$.

(c) Find the equation for the line normal to the surface $x^2 + xy^2 + xyz = 4$ at $(1, 1, 2)$.

(d) Find the points on the sphere $x^2 + y^2 + z^2 = 1$ where the tangent plane is parallel to the plane $2x + y - 3z = 2$.

(e) Find the points on the sphere $(x + 1)^2 + (y - 1)^2 + z^2 = 1$ where the tangent plane is parallel to the plane $2x + 2y - z = 1$. 
(7) Find the domain and first partial derivatives of the following functions.
   (a) \( f(s, t) = (s^2 + t^2) \sin(s^2 - t^2) \).
   (b) \( g(x, y) = \frac{2x-3y}{x+2y} \).
   (c) \( h(x, y) = \ln\left(\frac{x+y}{x-y}\right) \).
   (d) \( k(x, t) = \frac{(3x+4t)e^{(x^2-t^2)}}{x^2+t^2} \).

(8) (a) Verify that \( u = \frac{1}{\sqrt{x^2+y^2+z^2}} \) is a solution of \( u_{xx} + u_{yy} + u_{zz} = 0 \).
   (b) Show that \( v(x, t) = f(x+2t) + g(x-2t) \) is a solution of the wave equation \( v_{tt} = 4v_{xx} \).

(9) Use implicit differentiation to find \( z_x \) and \( z_y \) if \( xyz = e^{x^2+y^2+z^2} \).

(10) Suppose that over a certain region of plane the electrical potential is given by \( V(x, y) = x^2 - xy + y^2 \).
   (a) Find \( \nabla V(x, y) \).
   (b) Find the direction of the greatest decrease in the electrical potential at the point \((1, 1)\). What is the magnitude of the greatest decrease?
   (c) Find the direction of the greatest increase in the electrical potential at the point \((1, 1)\). What is the magnitude of the greatest increase?
   (d) Find a direction at the point \((1, 1)\) in which the temperature does not increase or decrease.
   (e) Find the rate of change of \( V \) at \((1, 1)\) in the direction \( \langle 3, -4 \rangle \).

(11) Find the local maxima, local minima and saddle points of the following functions. Decide if the local maxima or minima is global maxima or minima. Explain.
   (a) \( f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 \)
   (b) \( f(x, y) = x^2 + y^3 - 3xy \)
   (c) \( f(x, y) = xy + \ln(x) + y^2 - 10, \quad x > 0 \)

(12) Find rigorously the global maximum/minimum and global maximizer/minimizer of the following functions subject to the given constraint.
   (a) \( f(x, y) = x^2y^2 - 2x - 2y, \quad 0 \leq x \leq 1 \) and \( 0 \leq y \leq 1 \).
   (b) \( f(x, y) = x^2y^2 - 2x - 2y, \quad 0 \leq x, \quad 0 \leq y \) and \( x + y \leq 1 \).