# Review Problems for Midterm II

**MATH 2850 – 004**

The detail of the information about the second midterm can be found at
http://www.math.utoledo.edu/~mtsui/calc06sp/exam/midterm2.html
**You should also review the homework and quiz problems to prepare for the midterm.**
**This midterm will cover 13.7 , Chapter 14 and 15.1-15.7.**

(1) (a) Change each of the following points from rectangular coordinates to cylindrical coordinates and spherical coordinates:

$$(2, -1, 2), (2, -2, -3) .$$

(b) Convert the equation $\cos(\phi) = \sin(\theta)$ into rectangular coordinates.
(c) Convert the equation $r\cos(\theta) = z$ into rectangular coordinates.

(2) Find the arc-length of the curve $r(t) = \langle 2t, e^t, e^{-t}\rangle$ when $0 \le t \le \ln(2)$.

(3) (a) Find parametric equations for the tangent line to the curve $r(t) = \langle t^3, t, t^3\rangle$ at the point $(-1, 1, -1)$.
(b) At what point on the curve $r(t) = \langle t^3, t, t^3\rangle$ is the normal plane (this is the plane that is perpendicular to the tangent line) parallel to the plane $24x + 2y + 24z = 3$?

(4) Find the unit tangent, unit normal, binormal vectors and curvature of the curve $r(t) = \langle 4t, \cos(3t), \sin(3t)\rangle$.

(5) Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(1, 2, 2)$ and use it to estimate $\sqrt{(1.1)^2 + (2.1)^2 + (1.9)^2}$.

(6) (a) Find the equation for the plane tangent to the surface $z = 3x^2 - y^2 + 2x$ at $(1, -2, 1)$.

(b) Find the equation for the plane tangent to the surface $x^2 + xy^2 + xyz = 4$ at $(1, 1, 2)$.

(c) Find the equation for the line normal to the surface $x^2 + xy^2 + xyz = 4$ at $(1, 1, 2)$.

(d) Find the points on the sphere $x^2 + y^2 + z^2 = 1$ where the tangent plane is parallel to the plane $2x + y - 3z = 2$.
(e) Find the points on the sphere $(x + 1)^2 + (y - 1)^2 + z^2 = 1$ where the tangent plane is parallel to the plane $2x + 2y - z = 1$.

(7) Find the domain and first partial derivatives of the following functions.

(a) $f(s,t) = (s^2 + t^2)\sin(s^2 - t^2)$.

(b) $g(x,y) = \frac{2x-3y}{x+2y}$.

(c) $h(x,y) = \ln(\frac{x+y}{x-y})$.

(d) $k(x,t) = \frac{(3x+4t)e^{(x^2-t^2)}}{x^2+t^2}$.

(8) (a) Verify that $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ is a solution of $u_{xx} + u_{yy} + u_{zz} = 0$.

(b) Show that $v(x,t) = f(x+2t) + g(x-2t)$ is a solution of the wave equation $v_{tt} = 4v_{xx}$.

(9) Use implicit differentiation to find $z_x$ and $z_y$ if $xyz = e^{x^2+y^2+z^2}$.

(10) Suppose that over a certain region of plane the electrical potential is given by $V(x,y) = x^2 - xy + y^2$.

(a) Find $\nabla V(x,y)$.

(b) Find the direction of the greatest decrease in the electrical potential at the point $(1,1)$. What is the magnitude of the greatest decrease?

(c) Find the direction of the greatest increase in the electrical potential at the point $(1,1)$. What is the magnitude of the greatest increase?

(d) Find a direction at the point $(1,1)$ in which the temperature does not increase or decrease.

(e) Find the rate of change of $V$ at $(1,1)$ in the direction $\langle 3, -4\rangle$.

(11) Find the local maxima, local minima and saddle points of the following functions. Decide if the local maxima or minima is global maxima or minima. Explain.

(a) $f(x,y) = 3x^2y + y^3 - 3x^2 - 3y^2$

(b) $f(x,y) = x^2 + y^3 - 3xy$

(c) $f(x,y) = xy + \ln(x) + y^2 - 10$, $x > 0$

(12) Find rigorously the global maximum/minimum and global maximizer/minimizer of the following functions subject to the given constraint.

(a) $f(x,y) = x^2y^2 - 2x - 2y$, $0 \le x \le 1$ and $0 \le y \le 1$.

(b) $f(x,y) = x^2y^2 - 2x - 2y$, $0 \le x$, $0 \le y$ and $x + y \le 1$.