Review Problems for Midterm III
MATH 2850 – 004

The detail of the information about the second midterm can be found at http://www.math.utoledo.edu/~mtsui/calc06sp/exam/midterm3.html
You should also review the homework and quiz problems to prepare for the midterm.
This midterm will cover 15.8 Lagrange multipliers , Chapter 16 (except 16.9) and 17.1-17.2.

(1) Use Lagrange multipliers to find the maximum or minimum values of \( f \) subject to the given constraint.
(a) \( f(x, y) = xy, \ (1 + x^2)(1 + y^2) = 4 \).
(b) \( f(x, y, z) = x^2 - y^2, \ x^2 + y^2 = 2 \)
(c) \( f(x, y, z) = x + y + z, \ x^2 + y^2 + z^2 = 1 \).

(2) Using Riemann sums with two subdivisions in each direction, find upper and lower bounds for the volume under the graph of \( f(x, y) = 1 + x^2 + y^2 \) above the rectangle \( R \) with \( 0 \leq x \leq 2, 0 \leq y \leq 2 \).

(3) Compute the following iterated integrals.
(a) \( \int \int_D \frac{6x}{y+1} \, dA \) where \( D = \{(x, y)| 0 \leq x \leq y, 0 \leq y \leq 1\} \).
(b) \( \int_0^1 \int_{\sqrt{y}}^1 \frac{ye^2}{x^3} \, dxdy \)
(c) \( \int_0^1 \int_x^1 \cos(y^2) \, dydx \)
(d) \( \int_{-3}^3 \int_{0}^3 \sqrt{x^2 + y^2} \, dxdy \)
(e) \( \int_2^0 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} e^{-x^2-y^2} \, dydx \)
(f) \( \int_0^1 \int_{0}^1 \int_{x^2+y^2}^{x^2+y^2} xyzdzdx dy \)
(g) \( \int_1^1 \int_{\sqrt{x^2-y^2}}^{\sqrt{x^2+y^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{\frac{3}{2}} \, dzdy dx \)
(h) \( \int_2^0 \int_{\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} \, dzdx dy \)
(i) \( \int_3^0 \int_{\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_{\sqrt{18-x^2-y^2}}^{\sqrt{18-x^2-y^2}} (x^2 + y^2 + z^2) \, dzdx dy \)

(4) Find the volume of the following regions:
(a) The solid bounded by the surface \( z = x\sqrt{x^2 + y} \) and the planes \( x = 0, x = 1, y = 0, y = 1 \) and \( z = 0 \).
(b) The solid that lies between the sphere \( x^2 + y^2 + z^2 = 4 \), above the \( x - y \) plane, and below the cone \( z = \sqrt{x^2 + y^2} \).
(c) The solid bounded by the plane \( x + y + z = 3, x = 0, y = 0 \) and \( z = 0 \).
(d) The region bounded by the cylinder \( x^2 + y^2 = 4 \) and the plane \( z = 0 \) and \( y + z = 3 \).

(5) Find the area of the following surfaces.
(a) The cone \( z = \sqrt{x^2 + y^2} \) between the planes \( z = 1 \) and \( z = 2 \)
(b) Given by \( \{(x, y, z) \mid x^2 + y^2 = 1, 0 \leq z \leq xy\} \).

(6) Rewrite the integral \( \int_{-1}^{1} \int_{0}^{1} \int_{0}^{1-y} f(x, y, z)dzdydx \) as an iterated integral in the order of \( dx dy dz \)

(7) Evaluate the following line integrals.
(a) \( \int_C zds \) where \( C \) is given by \( x = \cos(t), y = \sin(t), z = t, 0 \leq t \leq \pi \).
(b) Evaluate \( \int_C \mathbf{F} \cdot dr \) where \( \mathbf{F} = \langle y, x \rangle \) and \( C \) is given by \( r(t) = (t, t^2) \) where \( 0 \leq t \leq 1 \).
(c) \( \int_C ydx + xdy \) where \( C \) is the line between \( A = (1, 1) \) where \( B = (-1, 5) \).
(d) \( \int_C \mathbf{F} \cdot dr \) where \( \mathbf{F} = \langle x, y, z \rangle \) and \( C \) is given by \( r(t) = (\cos(t), \sin(t), t) \) where \( 0 \leq t \leq \pi \).
(e) \( \int_C \mathbf{F} \cdot dr \) where \( \mathbf{F} = \langle x, y, z \rangle \) and \( C \) is the line between \( A = (1, 1, 1) \) where \( B = (-1, 0, 3) \).

(8) Sketch the gradient vector field of \( f(x, y) = -xy \).

(9) Sketch the gradient vector field of \( f(x, y, z) = -\frac{x^2}{2} \).

(10) Consider a thin plate that occupies the region \( D \) bounded by the parabola \( y = 1 - x^2, \) \( x = 1 \) and \( y = 1 \) in the first quadrant with density function \( \rho(x, y) = y \).
(a) Find the mass of the thin plate.
(b) Find the center of mass of the thin plate.