Review Problems for Midterm III

MATH 2850 – 004

The detail of the information about the second midterm can be found at http://www.math.utoledo.edu/∼mtsui/calc06sp/exam/midterm3.html

You should also review the homework and quiz problems to prepare for the midterm.

This midterm will cover 15.8 Lagrange multipliers , Chapter 16 (except 16.9) and 17.1-17.2.

(1) Use Lagrange multipliers to find the maximum or minimum values of f subject to the given constraint.
   (a) $f(x, y) = xy$, $(1 + x^2)(1 + y^2) = 4$.
   (b) $f(x, y, z) = x^2 - y^2$, $x^2 + y^2 = 2$
   (c) $f(x, y, z) = x + y + z$, $x^2 + y^2 + z^2 = 1$.

(2) Using Riemann sums with two subdivisions in each direction, find upper and lower bounds for the volume under the graph of $f(x, y) = 1 + x^2 + y^2$ above the rectangle R with $0 \le x \le 2$, $0 \le y \le 2$.

(3) Compute the following iterated integrals.
   (a) $\int\int_D \frac{6x}{y^3+1} dA$ where $D = \{(x, y)|0 \le x \le y, 0 \le y \le 1\}$.
   (b) $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dxdy$
   (c) $\int_0^1 \int_x^1 \cos(y^2) dydx$
   (d) $\int_{-3}^0 \int_0^{\sqrt{9-y^2}} \sqrt{x^2 + y^2} dxdy$
   (e) $\int_0^2 \int_{-\sqrt{4-x^2}}^0 e^{-x^2-y^2)} dydx$
   (f) $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz dzdxdy$
   (g) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} (x^2 + y^2)^{\frac{3}{2}} dzdydx$
   (h) $\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{-\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} dzdxdy$
   (i) $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2 + y^2 + z^2) dzdxdy$

(4) Find the volume of the following regions:
   (a) The solid bounded by the surface $z = x\sqrt{x^2 + y}$ and the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$ and $z = 0$.
   (b) The solid that lies between the sphere $x^2 + y^2 + z^2 = 4$, above the $x - y$ plane, and below the cone $z = \sqrt{x^2 + y^2}$.
   (c) The solid bounded by the plane $x + y + z = 3$, $x = 0$, $y = 0$ and $z = 0$.

(d) The region bounded by the cylinder $x^2+y^2=4$ and the plane $z=0$ and $y+z=3$.

(5) Find the area of the following surfaces.
 (a) The cone $z=\sqrt{x^2+y^2}$ between the planes $z=1$ and $z=2$
 (b) Given by $\{(x,y,z)|x^2+y^2=1, 0\leq z\leq xy\}$.

(6) Rewrite the integral $\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} f(x,y,z)dzdydx$ as an iterated integral in the order of $dxdydz$

(7) Evaluate the following line integrals.
 (a) $\int_C 2zds$ where C is given by $x=\cos(t)$, $y=\sin(t)$, $z=t$, $0\leq\pi$.
 (b) Evaluate $\int_C F\cdot dr$ where $F=<y,x>$ and C is given by $r(t)=(t,t^2)$ where $0\leq t\leq 1$.
 (c) $\int_C ydx+xdy$ where C is the line between $A=(1,1)$ where $B=(-1,5)$.
 (d) $\int_C F\cdot dr$ where $F=<x,y,z>$ and C is given by $r(t)=(\cos(t),\sin(t),t)$ where $0\leq t\leq\pi$.
 (e) $\int_C F\cdot dr$ where $F=<x,y,z>$ and C is the line between $A=(1,1,1)$ where $B=(-1,0,3)$.

(8) Sketch the gradient vector field of $f(x,y)=-xy$.

(9) Sketch the gradient vector field of $f(x,y,z)=-\frac{x^2}{2}$.

(10) Consider a thin plate that occupies the region D bounded by the parabola $y=1-x^2$, $x=1$ and $y=1$ in the first quadrant with density function $\rho(x,y)=y$.
 (a) Find the mass of the thin plate.
 (b) Find the center of mass of the thin plate.