Problem Set #1

1. Consider a molecule with four atoms at the points \((0, 0, 0), (6, 0, 0), (3, \sqrt{3}, 2\sqrt{6})\) and \((3, 3\sqrt{3}, 0)\). Verify that every atom in this molecule is 6 units away from every other atom.

Solution. If \(A\) is the point \((0, 0, 0)\), \(B\) is the point \((6, 0, 0)\), \(C\) is the point \((3, \sqrt{3}, 2\sqrt{6})\) and \(D\) is the point \((3, 3\sqrt{3}, 0)\), then we have

\[
\|\overrightarrow{AB}\| = \sqrt{(6 - 0)^2 + (0 - 0)^2 + (0 - 0)^2} = \sqrt{36} = 6 \\
\|\overrightarrow{AC}\| = \sqrt{(3 - 0)^2 + (\sqrt{3} - 0)^2 + (2\sqrt{6} - 0)^2} = \sqrt{9 + 3 + 24} = \sqrt{36} = 6 \\
\|\overrightarrow{AD}\| = \sqrt{(3 - 0)^2 + (3\sqrt{3} - 0)^2 + (0 - 0)^2} = \sqrt{9 + 27} = 6 \\
\|\overrightarrow{BC}\| = \sqrt{(3 - 6)^2 + (\sqrt{3} - 0)^2 + (2\sqrt{6} - 0)^2} = \sqrt{9 + 3 + 24} = \sqrt{36} = 6 \\
\|\overrightarrow{BD}\| = \sqrt{(3 - 3)^2 + (3\sqrt{3} - \sqrt{3})^2 + (0 - 2\sqrt{6})^2} = \sqrt{12 + 24} = \sqrt{36} = 6
\]

which establishes that every atom in this molecule is 6 units away from every other atom.

\(\square\)

2. Let \(P = (1, 2, 3)\) and \(Q = (3, 4, 2)\).

a Find the distance between \(P\) and \(Q\).

b Find a unit vector from the point \(P\) and toward the point \(Q\).

c Find a vector of length 9 pointing in the same direction of \(\overrightarrow{PQ}\).

d Find a point \(R\) such that \(\overrightarrow{PR}\) is a vector of length 12 pointing in the opposite direction of \(\overrightarrow{PQ}\).

Solution. The displacement vector \(\overrightarrow{PQ} = (3, 4, 2) - (1, 2, 3) = < 2, 2, -1 >\). The distance between \(P\) and \(Q = ||PQ|| = \sqrt{2^2 + 2^2 + (-1)^2} = \sqrt{9} = 3\).

The unit vector from the point \(P\) and toward the point \(Q\)

\[
= \frac{\overrightarrow{PQ}}{||PQ||} = \frac{1}{3}(2, 2, -1) = (\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}).
\]

The vector of length 9 pointing in the same direction of \(\overrightarrow{PQ}\)

\[
= 9 \frac{\overrightarrow{PQ}}{||PQ||} = 9(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}) = (6, 6, -3).
\]

Let \(R = (p, q, r)\) Then \(\overrightarrow{PR} = (p, q, r) - (1, 2, 3) = (p - 1, q - 2, r - 3)\). We need to solve \(\overrightarrow{PR} = -12 \frac{\overrightarrow{PQ}}{||PQ||}\). By (c), we have

\[
-12 \frac{\overrightarrow{PQ}}{||PQ||} = -12(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}) = (-8, -8, 4)
\]

Thus we have to solve \((p - 1, q - 2, r - 3) = (-8, -8, 4)\). So \(p = -7, q = -6\) and \(r = 7\). Hence \(R = (-7, -6, 7)\).

\(\square\)
3 Let $\vec{u} = <1, 1>$ and $\vec{v} = <-2, 1>$. Describe the set of vectors \( \{ \vec{w} = s\vec{u} + t\vec{v} | 0 \leq s \leq 1, 0 \leq t \leq 1 \} \) geometrically.

Solution. $\vec{w}$ is the set of vectors that start at origin and end at the parallelogram that spanned by the vectors $\vec{u} = <1, 1>$ and $\vec{v} = <-2, 1>$. 

![Diagram](image-url)