Problem Set #1

1. Consider a molecule with four atoms at the points $(0,0,0)$, $(6,0,0)$, $(3,\sqrt{3},2\sqrt{6})$ and $(3,3\sqrt{3},0)$. Verify that every atom in this molecule is 6 units away from every other atom.

Solution. If A is the point $(0,0,0)$, B is the point $(6,0,0)$, C is the point $(3,\sqrt{3},2\sqrt{6})$ and D is the point $(3,3\sqrt{3},0)$, then we have

$$\|\overrightarrow{AB}\| = \sqrt{(6-0)^2+(0-0)^2+(0-0)^2} = \sqrt{36} = 6$$
$$\|\overrightarrow{AC}\| = \sqrt{(3-0)^2+\left(\sqrt{3}-0\right)^2+\left(2\sqrt{6}-0\right)^2} = \sqrt{9+3+24} = \sqrt{36} = 6$$
$$\|\overrightarrow{AD}\| = \sqrt{(3-0)^2+\left(3\sqrt{3}-0\right)^2+(0-0)^2} = \sqrt{9+27} = \sqrt{36} = 6$$
$$\|\overrightarrow{BC}\| = \sqrt{(3-6)^2+\left(\sqrt{3}-0\right)^2+\left(2\sqrt{6}-0\right)^2} = \sqrt{9+3+24} = \sqrt{36} = 6$$
$$\|\overrightarrow{BD}\| = \sqrt{(3-6)^2+\left(3\sqrt{3}-0\right)^2+(0-0)^2} = \sqrt{9+27} = \sqrt{36} = 6$$
$$\|\overrightarrow{CD}\| = \sqrt{(3-3)^2+\left(3\sqrt{3}-\sqrt{3}\right)^2+\left(0-2\sqrt{6}\right)^2} = \sqrt{12+24} = \sqrt{36} = 6$$

which establishes that every atom in this molecule is 6 units away from every other atom. □

2 Let $P=(1,2,3)$ and $Q=(3,4,2)$.
a Find the distance between P and Q.
b Find a unit vector from the point P and toward the point Q.
c Find a vector of length 9 pointing in the same direction of $\overrightarrow{PQ}$.
d Find a point R such that $\overrightarrow{PR}$ is a vector of length 12 pointing in the opposite direction of $\overrightarrow{PQ}$.

Solution. The displacement vector $\overrightarrow{PQ} = (3,4,2)-(1,2,3) = <2,2,-1>$.
The distance between P and $Q = ||\overrightarrow{PQ}|| = \sqrt{2^2+2^2+(-1)^2} = \sqrt{9} = 3$.

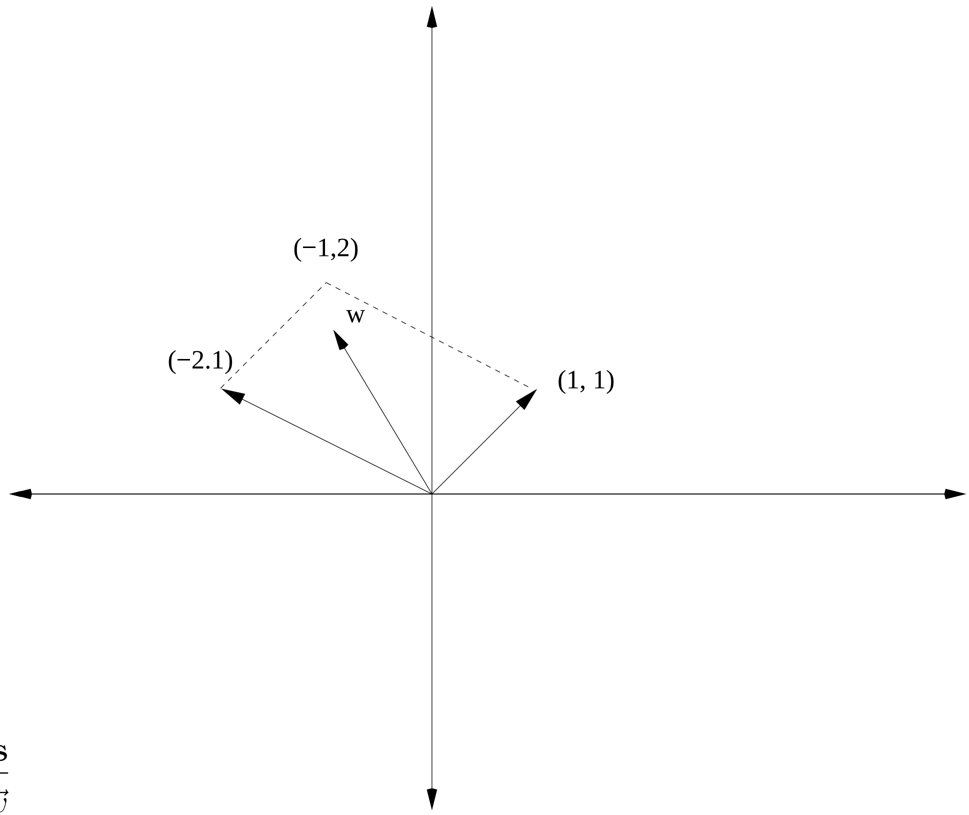
The unit vector from the point P and toward the point Q
$= \frac{\overrightarrow{PQ}}{||\overrightarrow{PQ}||} = \frac{1}{3}(2,2,-1) = (\frac{2}{3},\frac{2}{3},-\frac{1}{3})$.
The vector of length 9 pointing in the same direction of $\overrightarrow{PQ}$
$= 9\frac{\overrightarrow{PQ}}{||\overrightarrow{PQ}||} = 9(\frac{2}{3},\frac{2}{3},-\frac{1}{3}) = (6,6,-3)$.

Let $R=(p,q,r)$ Then $\overrightarrow{PR} = (p,q,r)-(1,2,3) = (p-1,q-2,r-3)$. We need to solve $\overrightarrow{PR} = -12\frac{\overrightarrow{PQ}}{||\overrightarrow{PQ}||}$. By (c), we have $-12\frac{\overrightarrow{PQ}}{||\overrightarrow{PQ}||} = -12(\frac{2}{3},\frac{2}{3},-\frac{1}{3}) = (-8,-8,4)$ Thus we have to solve $(p-1,q-2,r-3) = (-8,-8,4)$. So $p=-7$, $q=-6$ and $r=7$. Hence $R=(-7,-6,7)$.

□

3 Let $\vec{u} =< 1, 1 >$ and $\vec{v} =< -2, 1 >$.
Describe the set of vectors $\{\vec{w} = s\vec{u} + t\vec{v} | 0 \leq s \leq 1, 0 \leq t \leq 1\}$ geometrically.

Solution. $\vec{w}$ is the set of vectors that start at origin and end at the parallelogram that spanned by the vectors $\vec{u} =< 1, 1 >$ and $\vec{v} =< -2, 1 >$.



PSfrag replacements

$\vec{w}$

□