# Problem Set #5

**Due: Wednesday, Feb. 22**

**1.** Show that $v(x,y,t) = \frac{1}{t}e^{(\frac{-x^2-y^2}{4t})}$ is a solution of the heat equation $v_t = v_{xx} + v_{yy}$. (Hint: Find $v_t$, $v_{xx}$ and $v_{yy}$ first. Then verify $v_t - v_{xx} - v_{yy} = 0$.)

**2.** **(a)** Find an equation of the tangent plane to the surface $z = f(x,y) = \sqrt{x^2+y^2}$ at the point $(5,12,13)$

**(b)** Find the linear approximation of the function $f(x,y) = \sqrt{x^2+y^2}$ at $(5,12)$ and use it to estimate $\sqrt{(5.1)^2+(11.9)^2}$.

**3.** Let $z = f(x,y)$, $x = r\cos(\theta)$ and $y = r\sin(\theta)$.
Show that

$$(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = (\frac{\partial z}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial z}{\partial \theta})^2.$$

**4.** **(a)** Find equations of the tangent plane and the normal line to the surface $2x^2 = y^2 + z^2 - 12$ at the point $(2,-4,2)$.

**(b)** Find the points on the surface $2x^2 = y^2 + z^2 - 12$ where the tangent plane is parallel to the plane $4x + 4y + 2z = 1$.

**5.** The temperature at any point in the plane is given by the function

$$T(x,y) = \frac{10}{x^2+y^2+1}.$$

**(a)** Find $\nabla T(x,y)$.

**(b)** Find the direction of the greatest increase in temperature at the point $(3,4)$. What is the magnitude of the greatest increase?

**(c)** Find the direction of the greatest decrease in temperature at the point $(3,4)$. What is the magnitude of the greatest decrease?

**(d)** Find a direction at the point $(3,4)$ in which the temperature does not increase or decrease.

**(e)** Find the rate of change of $T$ at $(3,4)$ in the direction $\langle 5,12\rangle$.

**(f)** Find the directional derivative of $T$ at $(3,4)$ in the direction $\langle 4,3\rangle$.