Problem Set #7
Due: Wednesday, Mar. 22

1. (a) If $f(x, y)$ gives the pollution density, in micrograms per square meter, and $x$ and $y$ are measured in meters, give the units and practical interpretation of $\int \int_R f(x, y) \, dA$.

   (b) Using Riemann sums with two subdivisions in each direction, find upper and lower bounds for the volume under the graph of $f(x, y) = 2 + xy$ above the rectangle $R$ with $0 \leq x \leq 2$, $0 \leq y \leq 2$.

2. Find the volume of the solid bounded by the surface $z = y \sqrt{y^2 + x}$ and the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$ and $z = 0$.

3. Compute the following iterated integrals.

   (a) $\int_0^1 \int_0^1 \frac{xy}{\sqrt{x^2 + y^2 + 1}} \, dy \, dx$

   (b) $\int_0^1 \int_0^1 e^y \sqrt{x + e^y} \, dx \, dy$

4. Evaluate $\int \int_D \frac{4y}{x^2 + y^4} \, dA$ where $D = \{(x, y) \mid 1 \leq x \leq 6, 0 \leq y \leq 4x\}$.

5. Evaluate $\int \int_D x \sqrt{y^2 - x^2} \, dA$ where $D = \{(x, y) \mid 0 \leq x \leq y, 0 \leq y \leq 3\}$.

6. Evaluate $\int_0^1 \int_y^1 e^{x^2} \, dx \, dy$.

7. Evaluate $\int_0^1 \int_x^1 x^3 \sin(3y^3) \, dy \, dx$. 