Problem Set #8

Due: Wednesday, Mar. 29

1. Find the volume of an ice cream cone bounded by the hemisphere $z = \sqrt{8 - x^2 - y^2}$ and the cone $z = \sqrt{x^2 + y^2}$.

2. Evaluate the following integral by converting to polar coordinates.

(a) $\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2)^{\frac{3}{2}} dxdy$

(b) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sin(x^2 + y^2) dydx$

3. **(a)** For $a > 0$ find the volume under the graph of $z = e^{-(x^2+y^2)}$ above the disk $x^2 + y^2 \le a^2$.

(b) What happens to the volume as $a \to \infty$.

4. Consider a thin plate that occupies the region D bounded by the parabola $y = 1 - x^2$, $x = 0$ and $y = 0$ in the first quadrant with density function $\rho(x, y) = x$.

(a) Find the mass of the thin plate.

(b) Find the center of mass of the thin plate.

(c) Find moments of inertia I_x, I_y and I_0.

5. A joint density function is given by

$$f(x, y) = \begin{cases} kx^2 & \text{for } 0 \le x \le 2 \text{ and } 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant k.

(b) Find the probability that (x, y) satisfies $x + y \le 2$.

(c) Find the probability that (x, y) satisfies $x \le 1$ and $y \le 1/2$.