

MATH 2850 Solution to Quiz #7

1. $\int_0^1 \int_{y^2}^1 y^3 \cos(3x^3) dx dy$

Solution. Let $D = \{(x, y) | y^2 \leq x \leq 1, 0 \leq y \leq 1\}$. Since $y^2 \leq x$ and $0 \leq y$, we have $0 \leq y \leq \sqrt{x}$. Since $y^2 \leq x \leq 1$ and $0 \leq y$, we have $0 \leq x \leq 1$. So D is the same as $\{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq \sqrt{x}\}$.

$$\begin{aligned} \text{We have } \int_0^1 \int_{y^2}^1 y^3 \cos(3x^3) dx dy &= \int_0^1 \int_0^{\sqrt{x}} y^3 \cos(3x^3) dy dx = \int_0^1 \frac{y^4}{4} \cos(3x^3) \Big|_0^{\sqrt{x}} dx = \\ &= \int_0^1 \frac{x^2}{4} \cos(3x^3) dx = \frac{\sin(3x^3)}{36} \Big|_0^1 = \frac{\sin(3)}{36} - \frac{1}{36}. \end{aligned}$$

□

2. $\int_0^1 \int_y^1 e^{x^2} dx dy$.

Solution. Let $D = \{(x, y) | y \leq x \leq 1, 0 \leq y \leq 1\}$. Since $y \leq x \leq 1$ and $0 \leq y$, we have $0 \leq y \leq x$. Since $y \leq x \leq 1$ and $0 \leq y$, we have $0 \leq x \leq 1$.

So D is the same as $\{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq x\}$.

$$\begin{aligned} \text{We have } \int_0^1 \int_y^1 e^{x^2} dx dy &= \int_0^1 \int_0^x e^{x^2} dy dx = \int_0^1 ye^{y^2} \Big|_0^x dx = \int_0^1 e^{x^2} x dx = \frac{e^{x^2}}{2} \Big|_0^1 = \\ &= \frac{e}{2} - \frac{1}{2}. \end{aligned}$$

□