MATH 2850 Solution to Quiz #8

1. $\int_{-3}^{0}\int_{-\sqrt{9-y^2}}^{0}\sqrt{x^2+y^2}dxdy$

Solution. The region of integration is $\{(x,y)-\sqrt{9-y^2}\leq x\leq 0,-3\leq y\leq 0\}$. Since $-\sqrt{9-y^2}\leq x$, we have $x^2+y^2\leq 9$. Using $x\leq 0$ and $-3\leq y\leq 0$, we conclude that this region is in third quadrant. In polar coordinates, it is $R=\left\{(r,\theta):0\leq r\leq 3,\pi\leq\theta\leq\frac{3\pi}{2}\right\}$. We also have $\sqrt{x^2+y^2}=(r^2)^{\frac{1}{2}}=r$ and

$\int_{-3}^{0}\int_{-\sqrt{9-y^2}}^{0}\sqrt{x^2+y^2}dxdyx=\int_{\pi}^{\frac{3\pi}{2}}\int_{0}^{3}r\cdot rdrd\theta$

$=\int_{\pi}^{\frac{3\pi}{2}}\int_{0}^{3}\frac{r^3}{3}|_0^3d\theta=\frac{9\pi}{2})$.

□

2. $\int_R e^{x^2+y^2}dA$ where $R=\{(x,y)|x^2+y^2\leq 4\}$.

Solution. In polar coordinates, the region R is described by the inequalities $0\leq r\leq 2$, $0\leq\theta\leq 2\pi$ and the function is e^{r^2}. Hence

$$\int_R e^{x^2+y^2}dA=\int_0^2\int_0^{2\pi}e^{r^2}\,rdr\,d\theta=2\pi\int_0^2 re^{r^2}\,dr=2\pi\left[\frac{1}{2}e^{r^2}\right]_0^2=\pi\big(e^4-1)\big).$$

□