The following results will be given in the final exam.

(1) The substitution $v = y^{1-n}$ transforms the Bernoulli equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$ into the linear equation
$\frac{dv}{dx} + (1-n)P(x)v = (1-n)Q(x)$.

(2) The solution of the first order equation $y'(t) + p(t)y(t) = q(t)$ can be written as $(e^{\int p(t)dt}y)' = e^{\int p(t)}q(t)$.

(3) The equation $M(x,y)dx + N(x,y)dy = 0$ is exact if
$\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x}$.

(4) (The reduction of order) Suppose that $y_1(t)$ and $y_2(t)$ are solutions of the equation $y''(t) + p(t)y'(t) + q(t)y(t) = 0$.
Then $(\frac{y_2}{y_1})' = \frac{Ce^{-\int p(t)dt}}{y_1^2}$.

(5) (variation of parameter) Suppose $y_1(t)$ and $y_2(t)$ are independent solutions of $y''(t) + p(t)y'(t) + q(t)y(t) = 0$. Then a particular solution to $y''(t) + p(t)y'(t) + q(t)y(t) = g(t)$ is given by
$y_p(t) = -y_1(t)\int \frac{y_2(t)g(t)}{W(t)}dt + y_2(t)\int \frac{y_1(t)g(t)}{W(t)}dt$
where $W(t) = W(y_1,y_2)(t) = y_1(t)y_2'(t) - y_2(t)y_1'(t)$ is the Wronskian of y_1 and y_2.

(6) $(D-r)^k(t^l e^{rt}) = 0$ if $k > l$.

(7) $((D-a)^2+b^2)^k(t^l e^{at}\cos(bt)) = 0$ and $((D-a)^2+b^2)^k(t^l e^{at}\sin(bt)) = 0$ if $k > l$.

(8) $L(e^{at}) = \frac{1}{s-a}$, $L(1) = \frac{1}{s}$, $L(t^n) = \frac{n!}{s^{n+1}}$, $L(t^n e^{at}) = \frac{n!}{(s-a)^{n+1}}$,
$L(e^{at}\sin(bt)) = \frac{b}{(s-a)^2+b^2}$, $L(e^{at}\cos(bt)) = \frac{(s-a)}{(s-a)^2+b^2}$,
$L(u_c(t)) = \frac{e^{-cs}}{s}$, $L(\delta(t-c)) = e^{-cs}$, $L(u_c(t)f(t-c)) = e^{-cs}L(f(t))$,
$L^{-1}(e^{-cs}F(s)) = u_c(t)f(t-c)$ where $f(t) = L^{-1}(F(s))$,
$L(y^{(n)}(t)) = s^n L(y(t)) - s^{n-1}y(0) - \cdots - y^{(n-1)}(0)$.
$L(\int_0^t f(t-\tau)g(\tau)d\tau) = L(f(t))L(g(t))$.

$$u_c(t) = \begin{cases} 0, & 0 \le t < c, \\ 1, & c \le t. \end{cases}$$

$$u_{a,b}(t) = u_a(t) - u_b(t) = \begin{cases} 0, & 0 \le t < a, \\ 1, & a \le t < b, \\ 0, & b \le t. \end{cases}$$

(9) Suppose A is a 2×2 matrix with repeated eigenvalue λ and only one eigenvector $\overrightarrow{v}$. Then the general solution of $x' = Ax$ is
$x(t) = c_1 e^{\lambda t}\overrightarrow{v} + c_2(te^{\lambda t}\overrightarrow{v} + e^{\lambda t}\overrightarrow{w})$ where $\overrightarrow{w}$ satisfies the equation $(A - \lambda I)\overrightarrow{w} = \overrightarrow{v}$.

(10) A linear system is asymptotically stable if all the solutions converge, a linear system is stable if all the solutions remain bounded but do not converge and a linear system is unstable if some solutions diverge to infinity.