Review Problems for Midterm I

MATH 3860 – 001

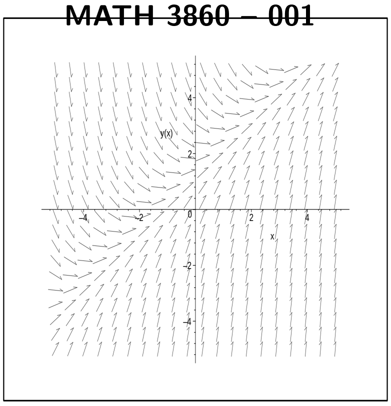

FIGURE 1. Slope fields of $\frac{dy}{dx} = (x - y + 1)(1 + \sin(xy)) + 1$

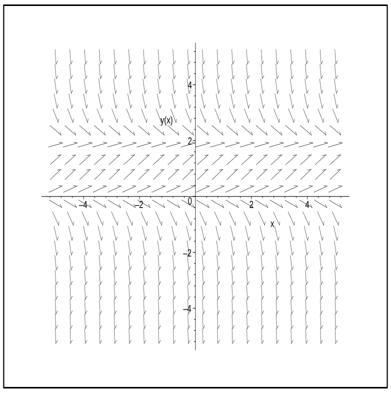

FIGURE 2. Slope fields of $\frac{dy}{dx} = (2y - y^2)(1 + x^2y^2)$

The information about the first midterm can be found at
http://www.math.utoledo.edu/∼mtsui/de05f/exam/midterm1.html

(1) The slope field of the indicated differential equation is given in below. Describe the behavior of the solution.
  (a) $\frac{dy}{dx} = (x - y + 1)(1 + \sin(xy)) + 1$.
  (b) $\frac{dy}{dx} = (2y - y^2)(1 + x^2y^2)$.

(2) (a) Show that the substitution $v = ax + by + c$ transforms the differential equation $\frac{dy}{dx} = F(ax + by + c)$ into a separable equation.
  (b) Use the idea above to solve the equation $\frac{dy}{dx} = (x + y + 1)^2$.

(3) (a) Show that the substitution $v = \frac{y}{x}$ transforms the differential equation $\frac{dy}{dx} = F(\frac{y}{x})$ into a separable equation.

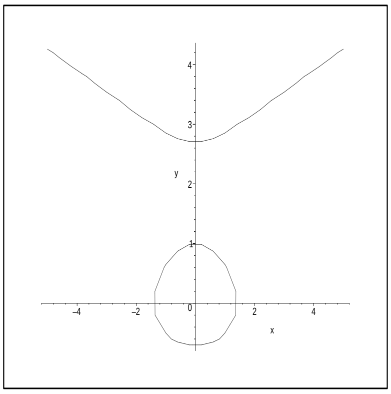

FIGURE 3. $y^3 - 3y^2 = x^2 - 2$

(b) Use the idea above to solve the equation $x^2\frac{dy}{dx} = y^2 + xy - x^2$.

(c) Show that $y = mx$ is a solution of $\frac{dy}{dx} = F(\frac{y}{x})$ if $m = F(m)$.

(4) (a) Solve the initial value problem

$$y' = \frac{2x}{3y^2 - 6y},\ y(0) = 1$$

and determine the interval where the solution is valid.

(b) Solve the initial value problem

$$y' = \frac{2x}{3y^2 - 6y},\ y(\sqrt{18}) = 4$$

and determine the interval where the solution is valid.

(5) Solve the following equations.

(a) $\frac{dy}{dt} + 3y = 2t + 1$. Can you say anything about $\lim_{t\to\infty} y(t)$?

(b) $\frac{dy}{dt} - y = \cos(t)$ with $y(0) = y_0$. Find the value of y_0 for which the solution remains finite as $t \to \infty$.

(c) $\frac{dy}{dt} - 3y = -e^{-t}$with $y(0) = y_0$. Find the value of y_0 for which the solution goes to $-\infty$ as $t \to \infty$. .

(d) $(t^2 + 1)\frac{dy}{dt} + 3ty = 6t$. Can you say anything about $\lim_{t\to\infty} y(t)$?

(e) $t^3 + 3y - t\frac{dy}{dt} = 0$.

(f) $y + t\frac{dy}{dt} = 2e^t$.

(g) $(2t + 1)\frac{dy}{dt} + y = (2t + 1)^{\frac{3}{2}}$.

(h) $t\frac{dy}{dt} = 6y + 12t^4y^{\frac{2}{3}}$. (Bernoulli equation).

(i) $4x^2y^2 + \frac{dy}{dx} = 5x^4y^2$.

(j) $6xy + 2y^2 + (9x^2 + 8xy + y)\frac{dy}{dx} = 0$.

(k) $e^y + y\cos(x) + (xe^y + \sin(x) + e^y)\frac{dy}{dx} = 0$.

(l) $\frac{dy}{dx} = (x + y)2 - 1$.

(m) $2x^2y - x^3\frac{dy}{dx} = y^3$.
(n) $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$.

(6) Show that the solution to $\frac{dy}{dt} = y^4$ with $y(0) = y_0 > 0$ escape to ∞ in a finite time.

(7) Without solving the problem, explain why the solution to $\frac{dy}{dt} = \frac{y\sin(t)}{1+t^2+y^2}$ with $y(0) = -2$ is always negative.

(8) Without solving the problem, determine the interval of existence of the following equation.
(a) $(9-t^2)\frac{dy}{dt} + \frac{y}{t} = \cos(t)$, $y(-1) = 2$.
(b) $(9-t^2)\frac{dy}{dt} + \frac{y}{t} = \cos(t)$, $y(1) = 2$.
(c) $(9-t^2)\frac{dy}{dt} + \frac{y}{t} = \cos(t)$, $y(4) = 2$.
(d) $(9-t^2)\frac{dy}{dt} + \frac{y}{t} = \cos(t)$, $y(-4) = 2$.

(9) In each problem, determine the equilibrium points, and classify each one as asymptotically stable, unstable, or semistable.
(a) $\frac{dy}{dt} = y^2\sin^2(y)$
(b) $\frac{dy}{dt} = y\sin(y)$
(c) $\frac{dy}{dt} = (-y^3 + 3y^2 - 2y)(y-3)^2$
(d) $\frac{dy}{dt} = y^3 - 3y^2 + 2y$

(10) Describe the behavior of the solution to the following differential equation
(a) $\frac{dy}{dt} = \frac{y^2(y-2)}{y-1}$
(b) $\frac{dy}{dt} = \frac{(y^2-4)}{y-1}$

(11) Without solving the differential equation, sketch the graph of the solution to $\frac{dy}{dt} = 4y - y^2$ with the following initial conditions. Also indicate the concavity of the solution.
(a) $y(0) = 8$
(b) $y(0) = 3$
(c) $y(0) = 1$
(d) $y(0) = -1$