Review Problems for Midterm II

MATH 3860 – 001

(1) Find the general solution of the following differential equations.
  (a) $y''(t) + 6y'(t) + 9y = 0$.
  (b) $y''(t) + 5y'(t) + 4y = 0$.
  (c) $y''(t) + 4y'(t) + 5y = 0$.
  (d) $t^2y''(t) + 7ty'(t) + 8y(t) = 0$.
  (e) $t^2y''(t) + 7ty'(t) + 10y(t) = 0$.
  (f) $t^2y''(t) + 5ty'(t) + 4y(t) = 0$.
  (g) $t^2y''(t) + ty'(t) + 9y = 0$. .
  (h) $ty''(t) + 2y'(t) = t^3 + t^2 + 1$.
  (i) $y''(t) + (y'(t))^3 = 0$.
  (j) $y''(t) = (t + y'(t))^2 - 1$.

(2) Find the solution of the following initial value problems.
  (a) $y''(t) + 4y'(t) + 5y = 0$, $y(0) = 1$ and $y'(0) = 3$.
  (b) $t^2y''(t) + 7ty'(t) + 10y(t) = 0$, $y(1) = 2$ and $y'(1) = -5$.

(3) In the following problems, a differential and one solution y_1 are given. Use the method of reduction of order to find the general solution solution.
  (a) $t^2y''(t) - t(t+2)y'(t) + (t+2)y(t) = 0; y_1(t) = t$.
  (b) $(t+1)y''(t) - (t+2)y'(t) + y(t) = 0; y_1(t) = e^t$.

(4) Find the general solution of the following differential equations.
  (a) $y''(t) + 5y'(t) + 6y(t) = e^t + \sin(t)$.
  (b) $y''(t) + 4y = 2\sin(2t) + 3\cos(t)$
  (c) $y''(t) + 4y = 4e^{4t}$
  (d) $y''(t) + 4y + 4y(t) = e^{-2t} + e^{2t}$
  (e) $y''(t) + 5y'(t) + 6y(t) = t^2 + 1$.
  (f) $t^2y''(t) - ty'(t) - 3y(t) = 4t^2$.
  (g) $y''(t) + 4y = \sec(2t)$
  (h) $y''(t) + 4y = \tan(2t)$
  (i) $t^2y''(t) - t(t+2)y'(t) + (t+2)y(t) = t^3e^t(1+t)$.
  Given that $y_1(t) = e^t$ is a solution of $t^2y''(t) - t(t+2)y'(t) + (t+2)y(t) = 0$.
  (j) $(1-t)y''(t) + ty'(t) - y(t) = 2(t-1)^2s^{-t}$.
  Given that $y_1(t) = t$ is a solution of $(1-t)y''(t) + ty'(t) - y(t) = 0$.

(5) Find the solution of the following initial value problems.

(a) $y''(t) + 4y = 2\sin(2t) + 3\cos(t)$, $y(0) = 3$ and $y'(0) = 5$. You may use the result in 4b.
(b) $y''(t) - ty'(t) + \sin(y(t)) = 0$, $y(1) = 0$ and $y'(1) = 0$.

(6) What is the form of the particular solution of the following equations? (You don't have to find the particular solution. For example, the form of the particular solution of $y'' + y = \sin(t)$ is $y_p(t) = ct\sin(t) + dt\sin(t)$.)
(a) $y''(t) + 4y = t^2e^{-4t}$
(b) $y''(t) + 4y + 4y(t) = te^{-2t} + e^{2t}$
(c) $y''(t) + 2y'(t) + 2y(t) = 2te^t\cos(t)$.
(d) $y''(t) + 2y'(t) + 2y(t) = 2te^t\sin(2t)$.
(e) $y''(t) + 4y = t\sin(2t) + 3t\cos(2t)$.

(7) Express the solution of the following equation in the form of $y = Ae^{Bt}\cos(Ct - D)$.

(a) $y''(t) + 2y'(t) + 2y(t) = 0$, $y(0) = 2$ and $y'(0) = 3$.
(b) $y''(t) + 4y'(t) + 5y(t) = 0$, $y(0) = 2$ and $y'(0) = 3$.

(8) Solve the following problems and describe the behavior of the solutions.
(a) $y''(t) + 4y(t) = A\cos(wt)$ if $w \neq 2$.
(b) $y''(t) + 4y(t) = A\cos(wt)$ if $w = 2$.