Review Problems for Midterm III

(1) Find the general solution of the following differential equations.
   (a) \( y^{(6)}(t) + 64y(t) = 0 \).
   (b) \( y^{(3)}(t) + 3y^{(2)}(t) + 2y'(t) = 0 \).
   (c) \( y^{(4)}(t) - 8y^{(2)}(t) + 16y = 0 \).
   (d) \( y^{(6)}(t) + 2y^{(3)}(t) + y(t) = 0 \).
   (e) \( (D^2 - 4D + 13)^2(D - 2)^2y(t) = 0 \).

(2) Use the method of Annihilators to find the form of particular solution of the following problems.
   (a) \( (D^3 - 2D^2 + D)y = t + \cos(t) + t\sin(t) + t^2e^t \).
   (b) \( (D^3 + D)y = t + \cos(t) + t\sin(t) + t^2e^t \).
   (c) \( y''(t) + 2y'(t) + 2y(t) = 3te^{-t}\cos(t) \).

(3) Use Laplace’s transform to find the solution of the following initial value problems.
   (a) \( y^{(3)}(t) - 3y^{(2)}(t) + 2y'(t) = e^{2t} \) with \( y(0) = 1, y'(0) = 0 \) and \( y''(0) = 0 \).
   (b) \( y''(t) + y(t) = \sin(2t) \) with \( y(0) = 0, y'(0) = 0 \).
   (c) \( y''(t) + 4y = g(t) \) with \( y(0) = 0 \) and \( y'(0) = 0 \) where
       \[
       g(t) = \begin{cases} 
       0, & 0 \leq t < 2, \\
       3(t - 2), & 2 \leq t < 4, \\
       6, & 4 \leq t. 
       \end{cases}
       \]
   (d) \( y''(t) + 4y'(t) + 4y = g(t) \) with \( y(0) = 0 \) and \( y'(0) = 0 \) where
       \[
       g(t) = \begin{cases} 
       0, & 0 \leq t < 2, \\
       3(t - 2), & 2 \leq t < 4, \\
       6, & 4 \leq t. 
       \end{cases}
       \]
   (e) \( y''(t) + 4y'(t) + 5y = g(t) \) with \( y(0) = 0 \) and \( y'(0) = 0 \) where
       \[
       g(t) = \begin{cases} 
       0, & 0 \leq t < 2, \\
       1, & 2 \leq t < 4, \\
       0, & 4 \leq t. 
       \end{cases}
       \]
   (f) \( y''(t) + 5y'(t) + 4y(t) = \delta(t - 2) \), with \( y(0) = 1 \) and \( y'(0) = 1 \).
   (g) \( y''(t) + 4y'(t) + 5y(t) = \delta(t - 2) \), with \( y(0) = 0 \) and \( y'(0) = 0 \).
   (h) \( y''(t) - 4y'(t) + 4y(t) = \delta(t - 2) \), with \( y(0) = 0 \) and \( y'(0) = 0 \).

(4) Express the solution of the given initial value problem in terms of the convolution integral.
   (a) \( y''(t) + 4y'(t) + 5y(t) = e^{2t}\cos(t) \), with \( y(0) = 0 \) and \( y'(0) = 0 \).
   (b) \( y''(t) - 2y'(t) + y(t) = te^t \), with \( y(0) = 0 \) and \( y'(0) = 0 \).
   (c) \( y''(t) - 3y'(t) + 2y(t) = te^t + te^{2t} \), with \( y(0) = 0 \) and \( y'(0) = 0 \).