MATH 3860 Solution to HW 2

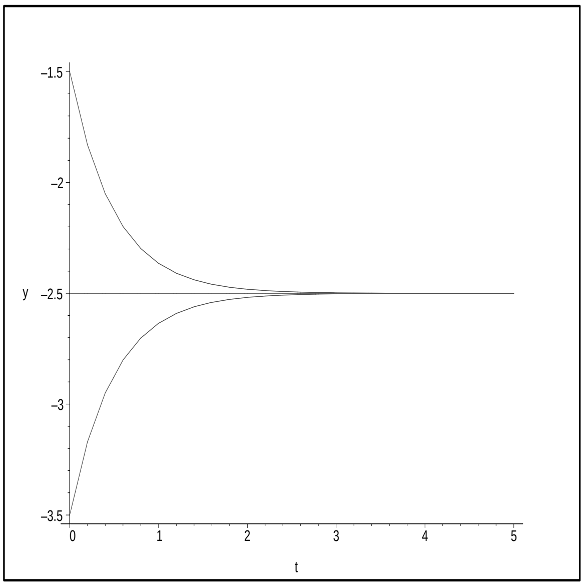

FIGURE 1.

1. (Solution to extra problem 1)

(a) It seems that that there is a linear solution. Assume $y(t) = mt + b$. We have $y'(t) = m$ and $t - y + 1 = t - (mt + b) + 1 = (1 - m)t - b + 1$. Hence $y'(t) = t - y + 1$ if $m = (1 - m)t - b$. The equation $m = (1 - m)t - b + 1$ can be simplified as $(1 - m)t - b - m + 1 = 0$. This implies that $1 - m = 0$ and $-b - m + 1 = 0$. Hence $m = 1$ and $b = 0$. So $y = t$ is a solution of $y'(t) = t - y + 1$.

(b) From the graph, it seems that every solution converges to the linear solution, i.e. $\lim_{t\to\infty} y(t) - t = 0$.

2. (Solution to extra problem 2) The solution of $y'(t) = y(1 - y)$ with $y(0) = y_0$ behaves in the following way.

(i) If $y_0 > 1$ then $\lim_{t\to\infty} y(t) = 1$.
(ii) If $y_0 = 1$ then $y(t) = 1$.
(iii) If $0 < y_0 < 1$ then $\lim_{t\to\infty} y(t) = 1$.
(vi) If $y_0 = 0$ then $y(t) = 0$.
(v) If $y_0 < 0$ then $\lim_{t\to\infty} y(t) = -\infty$.

3. (Sec1.2 (p15) 1b) From $\frac{dy}{dt} = -2y + 5$, we have $\int \frac{dy}{-2y+5} = \int dt$. So $\frac{\ln|-2y+5|}{-2} = t + c$, $-2y + 5 = Ce^{-2t}$ and $y = \frac{-5}{2} + Ce^{-2t}$. Using $y(0) = y_0$, we have $y_0 = \frac{-5}{2} + C$ and $C = y_0 + \frac{5}{2}$. So $y(t) = \frac{-5}{2} + (y_0 + \frac{5}{2})e^{-2t}$. We have $\lim_{t\to\infty} y(t) = \frac{-5}{2}$. See figure 1 for the graph of the solution.

From the graph, it seems that every solution converges to the linear solution, i.e. $\lim_{t\to\infty} y(t) - t = 0$.

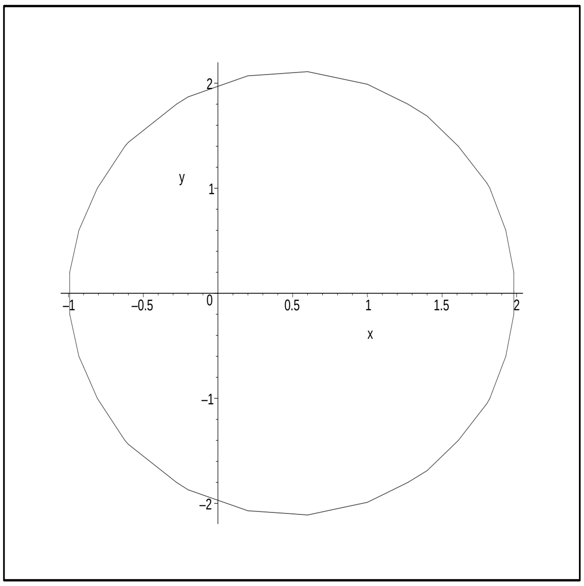

FIGURE 2. $\frac{y^2}{2} = x - x^2 + 2$

4. (Sec 2.2 Problem 10). From $\frac{dy}{dx} = \frac{(1-2x)}{y}$, we have $\int y dy = \int (1-2x) dx$. So $\frac{y^2}{2} = x - x^2 + c$. Using $y(1) = 2$, we get $2 = 1 - 1 + c$ and $c = 2$. The solution satisfies $\frac{y^2}{2} = x - x^2 + 2$. So $y^2 = 2(x - x^2 + 2)$ and $y(x) = \pm\sqrt{2(x - x^2 + 2)}$. Because $y(1) = 2$, we have $y = \sqrt{2(x - x^2 + 2)}$.

From $\frac{dy}{dx} = \frac{(1-2x)}{y}$, we know the solution doesn't exist when $y = 0$. Using $y^2 = 2(x - x^2 + 2)$ and $y = 0$, we have $x - x^2 + 2 = 0$. Solving $x - x^2 + 2 = 0$, we get $x = 2$ and $x = -1$. So the solution exists if $-1 < x < 2$.

5. (Sec 2.2 Problem 21). From $\frac{dy}{dx} = \frac{(1+3x^2)}{3y^2-6y}$, we have $\int 3y^2 - 6y dy = \int 1 + 3x^2 dx$. So $y^3 - 3y^2 = x + x^3 + c$. Using $y(0) = 1$, we get $1 - 3 = 0 + c$ and $c = -2$. The solution satisfies $y^3 - 3y^2 = x + x^3 - 2$.

From $\frac{dy}{dx} = \frac{(1+3x^2)}{3y^2-6y}$, we know the solution doesn't exist when $3y^2 - 6y = 3y(y-2) = 0$, i.e. $y = 0$ or $y = 2$. Using $y^3 - 3y^2 = x + x^3 - 2$ and $y = 0$, we have $x + x^3 - 2 = 0$. Factoring $x + x^3 - 2 = 0$, we get $(x-1)(x^2 + x + 2) = 0$. Thus $x = 1$ is the only real root of $x + x^3 - 2 = 0$.

Using $y^3 - 3y^2 = x + x^3 - 2$ and $y = 2$, we have $x + x^3 + 2 = 0$. Factoring $x + x^3 + 2 = 0$, we get $(x+1)(x^2 - x + 2) = 0$. Thus $x = -1$ is the only real root of $x + x^3 + 2 = 0$. So the solution exists if $-1 < x < 1$.

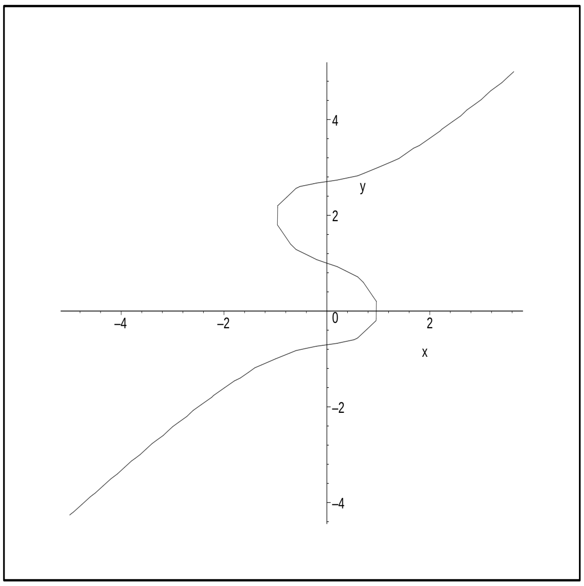

FIGURE 3. $y^3 - 3y^2 = x + x^3 - 2$