The step function $u_c(t)$ is defined by

$$u_c(t) = \begin{cases} 0, & 0 \leq t < c, \\ 1, & c \leq t, \end{cases}$$

The function $u_{a,b}(t)$ is defined by

$$u_{a,b}(t) = \begin{cases} 0, & 0 \leq t < a, \\ 1, & a \leq t < b, \\ 0, & b \leq t. \end{cases}$$

We have $u_{a,b}(t) = u_a(t) - u_b(t)$.

A function of the form

$$f(t) = \begin{cases} p(t), & 0 \leq t < a, \\ q(t), & a < t \end{cases}$$

can be expressed as the sum of step functions by noting that

$$f(t) = \begin{cases} p(t), & 0 \leq t < a, \\ q(t), & a \leq t, \end{cases}$$

$$= \begin{cases} p(t), & 0 \leq t < a, \\ 0, & a \leq t, \end{cases} + \begin{cases} 0, & 0 \leq t < a, \\ q(t), & a \leq t, \end{cases}$$

$$= p(t) \cdot \begin{cases} 1, & 0 \leq t < a, \\ 0, & a \leq t, \end{cases} + q(t) \cdot \begin{cases} 0, & 0 \leq t < a, \\ 1, & a \leq t, \end{cases}$$

$$= p(t)(u_0(t) - u_a(t)) + q(t)u_a(t)$$
$$= p(t)u_0(t) + (-p(t) + q(t))u_a(t).$$

A function of the form

$$f(t) = \begin{cases} p(t), & 0 \leq t < a, \\ q(t), & a \leq t < b, \\ r(t), & b \leq t. \end{cases}$$

can be expressed as the sum of step function by noting that

$$f(t) = \begin{cases} p(t), & 0 \leq t < a, \\ q(t), & a \leq t < b, \\ r(t), & b \leq t. \end{cases}$$

$$= \begin{cases} p(t), & 0 \leq t < a, \\ 0, & a \leq t < b, \\ 0, & b \leq t. \end{cases} + \begin{cases} 0, & 0 \leq t < a, \\ q(t), & a \leq t < b, \\ 0, & b \leq t. \end{cases} + \begin{cases} 0, & 0 \leq t < a, \\ 0, & a \leq t < b, \\ r(t), & b \leq t. \end{cases}$$

$$= p(t) \cdot \begin{cases} 1, & 0 \leq t < a, \\ 0, & a \leq t < b, \\ 0, & b \leq t. \end{cases} + q(t) \cdot \begin{cases} 0, & 0 \leq t < a, \\ 1, & a \leq t < b, \\ 0, & b \leq t. \end{cases} + r(t) \cdot \begin{cases} 0, & 0 \leq t < a, \\ 0, & a \leq t < b, \\ 1, & b \leq t. \end{cases}$$

$$= p(t)(u_0(t) - u_a(t)) + q(t)(u_a(t) - u_b(t)) + r(t)u_b$$
$$= p(t)u_0(t) + (-p(t) + q(t))u_a(t) + (-q(t) + r(t))u_b(t).$$