Review Problems for Midterm I

MATH 3860 – 002

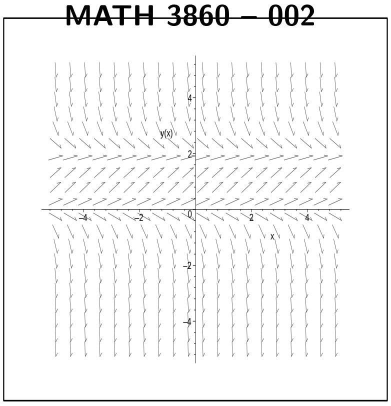

FIGURE 1. Slope fields of $\frac{dy}{dx} = (2y - y^2)(1 + x^2y^2)$

The information about the first midterm can be found at
http://www.math.utoledo.edu/∼mtsui/de06f/exam/midterm1.html

(1) The slope field of the indicated differential equation is given in Figure 1. Describe the behavior of the solution.
$\frac{dy}{dx} = (2y - y^2)(1 + x^2y^2)$.

(2) (a) Show that the substitution $v = ax + by + c$ transforms the differential equation $\frac{dy}{dx} = F(ax + by + c)$ into a separable equation in x and v. (Hint: Express the given equation as a differential equation in v.)
(b) Use the idea above to solve the equation $\frac{dy}{dx} = (x + y + 1)^2$.

(3) (a) Show that the substitution $v = \frac{y}{x}$ transforms the differential equation $\frac{dy}{dx} = F(\frac{y}{x})$ into a separable equation.
(b) Use the idea above to solve the equation $x^2\frac{dy}{dx} = y^2 + xy - x^2$.

(4) (a) Solve the initial value problem

$$y' = \frac{2x}{3y^2 - 6y},\ y(0) = 1$$

and determine the interval where the solution is valid. (You can use Figure 2 on page 2 to do this problem.)
(b) Solve the initial value problem

$$y' = \frac{2x}{3y^2 - 6y},\ y(\sqrt{18}) = 4$$

and determine the interval where the solution is valid. (You can use Figure 2 on page 2 to do this problem.)

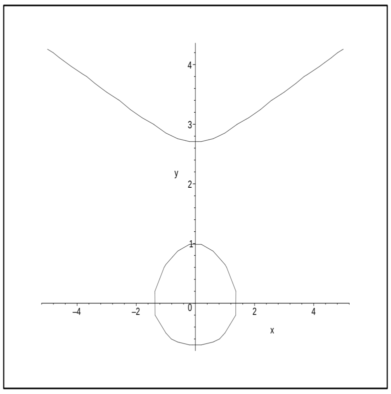

FIGURE 2. $y^3 - 3y^2 = x^2 - 2$

(5) Solve the following equations.
  (a) $\frac{dy}{dt} + 3y = 2t + 1$. Can you say anything about $\lim_{t\to\infty} y(t)$?
  (b) $\frac{dy}{dt} - 3y = -e^{-t}$with $y(0) = y_0$. Find the value of y_0 for which the solution goes to $-\infty$ as $t \to \infty$. .
  (c) $t^3 + 3y - t\frac{dy}{dt} = 0$.
  (d) $(2t + 1)\frac{dy}{dt} + y = (2t + 1)^{\frac{3}{2}}$.
  (e) $t\frac{dy}{dt} = 6y + 12t^4 y^{\frac{2}{3}}$. (Bernoulli equation. Try $v = y^{1-\frac{2}{3}}$.)
  (f) $4x^2y^2 + \frac{dy}{dx} = 5x^4y^2$.
  (g) $6xy^3 + 2y^4 + (9x^2y^2 + 8xy^3 + y^3)\frac{dy}{dx} = 0$.
  (h) $e^y + y\cos(x) + (xe^y + \sin(x) + e^y)\frac{dy}{dx} = 0$.
  (i) $2x^2y - x^3\frac{dy}{dx} = y^3$. (Bernoulli equation. Try $v = y^{1-3}$.)

(6) Without solving the problem, explain why the solution to $\frac{dy}{dt} = \frac{y\sin(t)}{1+t^2+y^2}$ with $y(0) = -2$ is always negative.

(7) Without solving the problem, determine the interval of existence of the following equation.
  (a) $(9 - t^2)\frac{dy}{dt} + \frac{y}{t} = \cos(t)$, $y(-1) = 2$.
  (b) $(9 - t^2)\frac{dy}{dt} + \frac{y}{t} = \cos(t)$, $y(1) = 2$.
  (c) $(9 - t^2)\frac{dy}{dt} + \frac{y}{t} = \cos(t)$, $y(4) = 2$.
  (d) $(9 - t^2)\frac{dy}{dt} + \frac{y}{t} = \cos(t)$, $y(-4) = 2$.

(8) In each problem, determine the equilibrium points, and classify each one as asymptotically stable, unstable, or semistable.
  (a) $\frac{dy}{dt} = y^2\sin^2(y)$
  (b) $\frac{dy}{dt} = y\sin(y)$
  (c) $\frac{dy}{dt} = (-y^3 + 3y^2 - 2y)(y - 3)^2$
  (d) $\frac{dy}{dt} = y^3 - 3y^2 + 2y$