Review Problems for Midterm II

(1) Find the interval of existence of the following initial value problems.
   (a) $(t^2-16)y''(t)+ty'(t)+\frac{9}{t-3}y=0$, $y(2)=1$ and $y'(2)=2$.
   (b) $(t^2-16)y''(t)+ty'(t)+\frac{9}{t-3}y=0$, $y(-1)=1$ and $y'(-1)=2$

(2) (a) Find the Wronskian of two solutions of the given differential equation without solving the equation. $x^2y''(x)+xy'(x)+x^2y(x)=0$.
   (b) If y_1 and y_2 are two independent solutions of $x^2y''(x)+xy'(x)+x^2y(x)=0$ and $W(y_1,y_2)(1)=5$. Find $W(y_1,y_2)(t)$.

(3) Find the general solution of the following differential equations.
   (a) $y''(t)+6y'(t)+9y=0$.
   (b) $y''(t)+5y'(t)+4y=0$.
   (c) $y''(t)+4y'(t)+5y=0$.
   (d) $t^2y''(t)+7ty'(t)+8y(t)=0$.
   (e) $t^2y''(t)+7ty'(t)+10y(t)=0$.
   (f) $t^2y''(t)+5ty'(t)+4y(t)=0$.
   (g) $t^2y''(t)+ty'(t)+9y=0$. .

(4) Find the solution of the following initial value problems.
   (a) $y''(t)+4y'(t)+5y=0$, $y(0)=1$ and $y'(0)=3$.
   (b) $t^2y''(t)+7ty'(t)+10y(t)=0$, $y(1)=2$ and $y'(1)=-5$.

(5) In the following problems, a differential and one solution y_1 are given. Use the method of reduction of order to find the general solution solution.
   (a) $t^2y''(t)-t(t+2)y'(t)+(t+2)y(t)=0; y_1(t)=t$.
   (b) $(t+1)y''(t)-(t+2)y'(t)+y(t)=0; y_1(t)=e^t$.

(6) Find the general solution of the following differential equations.
   (a) $y''(t)+5y'(t)+6y(t)=e^t+\sin(t)$.
   (b) $y''(t)+4y=2\sin(2t)+3\cos(t)$
   (c) $y''(t)+4y=4e^{4t}$
   (d) $y''(t)+4y+4y(t)=e^{-2t}+e^{2t}$
   (e) $y''(t)+5y'(t)+6y(t)=t^2+1$.
   (f) $t^2y''(t)-ty'(t)-3y(t)=4t^2$.
   (g) $y''(t)+4y=\sec(2t)$
   (h) $y''(t)+4y=\tan(2t)$
   (i) $t^2y''(t)-t(t+2)y'(t)+(t+2)y(t)=t^4e^t(1+t)$.
   Given that $y_1(t)=te^t$ is a solution of $t^2y''(t)-t(t+2)y'(t)+(t+2)y(t)=0$.
   (j) $(1-t)y''(t)+ty'(t)-y(t)=2(t-1)^2e^{-t}$.
   Given that $y_1(t)=t$ is a solution of $(1-t)y''(t)+ty'(t)-y(t)=0$.