Review Problems for Midterm III

(1) Find the general solution of the following differential equations.

(a) $y^{(6)}(t) + 64y(t) = 0.$

(b) $y^{(3)}(t) + 3y^{(2)}(t) + 2y'(t) = 0.$

(c) $y^{(4)}(t) - 8y^{(2)}(t) + 16y = 0.$

(d) $y^{(6)}(t) + 2y^{(3)}(t) + y(t) = 0.$

(e) $(D^2 - 4D + 13)^2(D-2)^2 y(t) = 0.$

(2) Use the method of Annihilators to find the form of particular solution of the following problems.

(a) $(D^3 - 2D^2 + D)y = t + \cos(t) + t\sin(t) + t^2e^t.$

(b) $(D^3 + D)y = t + \cos(t) + t\sin(t) + t^2e^t.$

(c) $y''(t) + 2y'(t) + 2y(t) = 3te^{-t}\cos(t).$

(3) Use Laplace's transform to find the solution of the following initial value problems.

(a) $y^{(3)}(t) - 3y^{(2)}(t) + 2y'(t) = e^{4t}$ with $y(0) = 1$, $y'(0) = 0$ and $y''(0) = 0$.

(b) $y''(t) + y(t) = \sin(2t)$ with $y(0) = 0$, $y'(0) = 0$.

(c) $y''(t) + 4y = g(t)$ with $y(0) = 0$ and $y'(0) = 0$ where

$$g(t) = \begin{cases} 0, & 0 \le t < 2, \\ 3(t-2), & 2 \le t < 4, \\ 6, & 4 \le t. \end{cases}$$

(d) $y''(t) + 5y'(t) + 4y = g(t)$ with $y(0) = 0$ and $y'(0) = 0$ where

$$g(t) = \begin{cases} 0, & 0 \le t < 2, \\ 3(t-2), & 2 \le t < 4, \\ 6, & 4 \le t. \end{cases}$$

(e) $y''(t) + 4y'(t) + 5y = g(t)$ with $y(0) = 0$ and $y'(0) = 0$ where

$$g(t) = \begin{cases} 0, & 0 \le t < 2, \\ 1, & 2 \le t < 4, \\ 0, & 4 \le t. \end{cases}$$

(f) $y''(t) + 5y'(t) + 4y(t) = \delta(t-2)$, with $y(0) = 1$ and $y'(0) = 1$.

(g) $y''(t) + 4y'(t) + 5y(t) = \delta(t-2)$, with $y(0) = 0$ and $y'(0) = 0$.

(h) $y''(t) - 4y'(t) + 4y(t) = \delta(t-2)$, with $y(0) = 0$ and $y'(0) = 0$.

(4) Express the solution of the given initial value problem in terms of the convolution integral.

(a) $y''(t) + 4y'(t) + 5y(t) = e^{2t}\cos(t)$, with $y(0) = 0$ and $y'(0) = 0$.

(b) $y''(t) - 2y'(t) + y(t) = te^t$, with $y(0) = 0$ and $y'(0) = 0$.

(c) $y''(t) - 3y'(t) + 2y(t) = te^t + te^{2t}$, with $y(0) = 0$ and $y'(0) = 0$.