

## MATH 3860 Solution to HW 1

1. (Solution to extra problem 1) The solution of  $y'(t) = y(1 - y)$  with  $y(0) = y_0$  behaves in the following way.
  - (i) If  $y_0 > 1$  then  $\lim_{t \rightarrow \infty} y(t) = 1$ .
  - (ii) If  $y_0 = 1$  then  $y(t) = 1$ .
  - (iii) If  $0 < y_0 < 1$  then  $\lim_{t \rightarrow \infty} y(t) = 1$ .
  - (vi) If  $y_0 = 0$  then  $y(t) = 0$ .
  - (v) If  $y_0 < 0$  then  $\lim_{t \rightarrow \infty} y(t) = -\infty$ .
  
2. Draw the directional field for the differential equation  $y'(t) = y(y - 2)$  at the following points  $(0, 1), (1, 1), (2, 1), (1, 2), (2, 2), (3, 2)$  on the t-y plane.

Solution: Please see figure 1.
  
3. Draw the directional field for the differential equation  $y'(t) = (t+1)(y-2)$  at the following points  $(0, 1), (1, 1), (2, 1), (1, 2), (2, 2), (3, 2)$  on the t-y plane.

Solution: Please see figure 2.
  
4. (Problem 5 from sec 1.3)  $\frac{d^2 y}{dt^2} + \sin(t+y) = \sin(t)$  is a second order O.D.E.
5. (Problem 6 from sec 1.3)  $\frac{d^3 y}{dt^3} + t \frac{dy}{dt} + \cos^2(t)y = t^3$  is a third order O.D.E.
6. (Problem 9 from sec 1.3)  $ty'(t) - y = t^2$  is a first order O.D.E.
7. (Problem 12 from sec 1.3)  $t^2 y''(t) + 5ty' + 4y = 0$  is a second order O.D.E.