MATH 3860 Solution to HW 3

1. (15 pts) (Problem 1 from sec 2.4) We can rewrite $(t-3)y' + \ln(t)y = 2t$ as $y' + \frac{\ln(t)}{t-3}y = \frac{2t}{(t-3)}$. We know that $p(t) = \frac{\ln(t)}{t-3}$ is continuous for $t \in (0,3) \cup (3,\infty)$ and $g(t) = \frac{2t}{(t-3)}$ is continuous for $t \in (-\infty,0) \cup (0,3) \cup (3,\infty)$. So $\frac{\ln(t)}{t-3}$ and $\frac{2t}{(t-3)}$ are continuous for $t \in (0,3) \cup (3,\infty)$. The initial value is $y(1) = 2$ and $1 \in (0,3)$. The solution exists if $t \in (0,3)$.

2. (15 pts) (Problem 4 from sec 2.4) We can rewrite $(4-t^2)y' + 2ty = 3t^2$ as $y' + \frac{2t}{4-t^2}y = \frac{3t^2}{(4-t^2)}$. We know that $p(t) = \frac{2t}{4-t^2} = \frac{2t}{(2-t)(2+t)}$ is continuous if $t \in (-\infty,-2) \cup (-2,2) \cup (2,\infty)$ and $g(t) = \frac{3t^2}{(4-t^2)}$ is continuous for $t(-\infty,-2) \cup (-2,2) \cup (2,\infty)$. So both $\frac{2t}{4-t^2}$ and $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$ are continuous for $t \in (-\infty,-2) \cup (-2,2) \cup (2,\infty)$. The initial value is $y(-3) = 2$ and $-3 \in (-\infty,-2)$. The solution of this problem exists if $t \in (-\infty,-2)$.

3. (15 pts) (Problem 5 from sec 2.4) The initial value is $y(1) = -3$ and $1 \in (-2,2)$. The solution of this problem exists if $t \in (-2,2)$.

4. (15 pts) (Problem 7 from sec 2.4) $y'(t) = \frac{t-y}{2t+5y}$.
Let $f(t,y) = \frac{t-y}{2t+5y}$. Then $\frac{\partial f}{\partial y} = \frac{-(2t+5y)-(t-y)\cdot 5}{(2t+5y)^2}$. So both $f(t,y)$ and $\frac{\partial f}{\partial y}$ are continuous in the region $\{(t,y) | 2t+5y \neq 0\}$.

5. (15 pts) (Problem 8 from sec 2.4) $y'(t) = (1-t^2-y^2)^{\frac{1}{2}}$.
Let $f(t,y) = (1-t^2-y^2)^{\frac{1}{2}}$. Then $\frac{\partial f}{\partial y} = \frac{-y}{(1-t^2-y^2)^{\frac{1}{2}}}$. So both $f(t,y)$ and $\frac{\partial f}{\partial y}$ are continuous in the region $\{(t,y) | 1-t^2-y^2 > 0\}$.

6. (25 pts)(Problem 28 from sec 2.4) Rewrite the equation $t^2y' + 2ty - y^3 = 0$ as $y' + \frac{2}{t}y - \frac{y^3}{t^2} = 0$.
Let $v = y^{-2}$. Then $\frac{dv}{dt} = -2y^{-3}\frac{dy}{dt} = -2y^{-3}(-\frac{2}{t}y + \frac{y^3}{t^2}) = \frac{4}{t}y^{-2} + \frac{2}{t^2} = \frac{4}{t}v + \frac{2}{t^2}$.
So $\frac{dv}{dt} - \frac{4}{t}v = \frac{2}{t^2}$. The integrating factor is $\mu(t) = e^{\int -\frac{4}{t}dt} = e^{-4\ln t} = t^{-4}$.
The solution of $\frac{dv}{dt} - \frac{4}{t}v = \frac{2}{t^2}$ is $v(t) = \frac{\int \mu(t)\frac{2}{t^2}}{\mu(t)} = \frac{\int t^{-4}\cdot\frac{2}{t^2}dt}{t^{-4}} = \frac{\int 2t^{-6}dt}{t^{-4}} = \frac{\frac{2t^{-5}}{-5}+C}{t^{-4}} = -\frac{2t^{-1}}{5} + Ct^4$.
Recall that $v = y^{-2}$. We have $y^2 = \frac{1}{v}$ and $y = \pm\sqrt{\frac{1}{v}} = \pm\sqrt{\frac{1}{-\frac{2t^{-1}}{5}+Ct^4}}$.