Problem Set #2

Due: Monday, October 9

1. Prove Lemma 3.5

2. Suppose M and N are smooth manifolds with M connected, and $F : M \mapsto N$ is a smooth map such that $F_* : T_pM \mapsto T_{F(p)}N$ is the zero map for each $p \in M$. Show that F is a constant map. (Hint: Show that F is locally constant by using coordinate representation of F_*.)

3. Let $F : S^3 \mapsto S^2$ defined by

$$F(x_1, x_2, x_3, x_4) = (2x_1x_3 + 2x_2x_4, 2x_2x_3 - 2x_1x_4, x_1^2 + x_2^2 - x_3^2 - x_4^2).$$

Find the coordinate representation of F_* near $(0, 0, 0, 1)$ in graph coordinates.

4. Let (w, x, y, z) be the coordinates in R^4. Consider the vector fields

$$V = -x\frac{\partial}{\partial w} + w\frac{\partial}{\partial x} - z\frac{\partial}{\partial y} + y\frac{\partial}{\partial z}$$

and

$$W = -y\frac{\partial}{\partial w} + z\frac{\partial}{\partial x} + w\frac{\partial}{\partial y} - x\frac{\partial}{\partial z}.$$

(a) Compute the Lie bracket $[V, W]$.

(b) Let $V|_{S^3}$ be the restriction of V to S^3 and $W|_{S^3}$ be the restriction of W to S^3. Compute the coordinate representations of $V|_{S^3}$ in graph coordinates and stereographic coordinates. Show that V is a smooth vector field on S^3.

(c) Compute the Lie bracket $[V|_{S^3}, W|_{S^3}]$.