Review Problem for Final Exam

Final Exam: Wednesday, Dec.11. 10:15-12:15
Topics: 65% of the final exam will be from 12.1–12.5 and 9.4
35% of the final exam will be from 5.8 and 6.1-6.3,7.1-7.4, 8.1-8.2
Office Hours before the exam: Friday (Dec 6th) 2-4pm M, Monday (Dec 9) 3-5pm and Tuesday (Dec 10) 3-5 pm. Email me to make appointment if these times are not good for you.
You may use the calculator in the exam.
The following formulae will be given in the exam.

a Bayes formula: Let B_1, B_2, $\cdots$, B_n form a partition of Ω, and let A be an event. Then $P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{j=1}^{n} P(A|B_j)P(B_j)}$

b Let S_n be a random variable that counts the number of successes in n independent trials, each having probability p of success. Then S_n is binomially distributed with parameters n and p, and $P(S_n = k) = \frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$, $k = 0, 1, 2, \cdots ., n$, $E(S_n) = np$, $Var(S_n) = np(1-p)$.

c A random variable X that counts the number of trials until the first success. The random variable X takes on values $1, 2, \cdots ., n$. Its distribution is called he geometric distribution and is given by $P(X = k) = (1-p)^k p$, $k = 1, 2, \cdots ., n$. Also, $P(X > k) = (1-p)^k$, $E(X) = \frac{1}{p}$ and $Var(X) = \frac{1-p}{p^2}$.

d X is Poisson distributed with parameter $\lambda > 0$ if $P(X = k) = e^{-\lambda}\frac{\lambda^k}{k!}$, $k = 0, 1, 2, \cdots$, $E(X) = \lambda$ and $var(X) = \lambda$

e If X is normally distributed with mean μ and standard deviation σ. 68-95-98 rule is that $P(X \in [\mu-\sigma, \mu+\sigma]) = 0.68$, $P(X \in [\mu-2\sigma, \mu+2\sigma]) = 0.95$ and $P(X \in [\mu-3\sigma, \mu+3\sigma]) = 0.99$. Suppose that Y is the standard normal distribution. Then $P(a \le X \le b) = P(\frac{a-\mu}{\sigma} \le Y \le \frac{b-\mu}{\sigma})$

1. Suppose that the probability mass function of a discrete random variable X is given by the following table:

x	0	1	2	3
P(X = x)	0.1	0.3	0.4	0.2

(a) Find $E(X)$. (b) Find $E(X^2)$ (c) $Var(X)$ (d) Find $E(2X-1)$.

2. A screening test for a disease shows a positive result in 95% of all cases when the disease is actually present and in 5% of all cases when it is not. Assume that the prevalence of the disease in the population is $1/50$.

(a) If the test is administered to a randomly chosen individual, what is the probability that the result is positive?

(b) Find the probability that person has the disease when the test is positive.

(c) Find the probability that person has the disease when the test is negative.
(d) Find the probability that person doesn't have the disease when the test is negative.

3. (12.4) Assume that 10% of all plants in a field are infested with aphids. Suppose that you pick 20 plants at random.
(a) Compute the expected value and the variance of the number of plants in a field are infested with aphids.
(b) What is the probability that none of them carried aphids?
(c) What is the probability that at least two of them carried aphids?
(d) What is the probability that at most three of them carried aphids?
(Hint: This is related to Binomial distribution.)

4. (12.4) A random experiment consists of flipping a fair coin until the first time heads appears.
(a) Find the probability that the first heads appears on the forth trial.
(b) Find the probability that the first heads appears after the forth trial.

5. (12.4) A random experiment consists of rolling a fair die until the first time a five or a six appears. (a) Find the probability that the first five or six appears on the third trial.
(b) Find the probability that the first heads appears after the third trial.

6. (12.4) The number of amino acid substitutions on a given amino acid sequence is Poisson distributed with mean 3. What is the probability of at least two substitutions?

7. (12.5) Use 65-95-98 rule to solve this problem. Assume that a quantitative character is normally distributed with mean μ and standard deviation σ . Determine what fraction of the population falls into the given interval.
(a) $[\mu, \infty)$ (b) $[\mu - 2\sigma, \mu + \sigma]$ (c) $(-\infty, \mu + 3\sigma]$
(d) $[\mu + \sigma, \mu + 2\sigma$ (e) $(-\infty, \mu - 2\sigma]$ (f) $[\mu - 3\sigma, \mu]$.

8. (12.5) Use the table on last page to do this problem. Assume that the mathematics score X on the Scholastic Aptitude Test (SAT) is normally distributed with mean 500 and standard deviation 100. (a) Find the probability that an individuals score exceeds 650.
(b) Find the probability that an individuals score is between 450 and 650.
(c) Find the math SAT score so that 10% of the students who took the

test have that score or greater.
(d) Find the math SAT score so that 75% of the students who took the test have that score or lower.

9. (9.4) Find the equation of the line through $(1,-2)$ and perpendicular to $[4,1]'$?

10. (9.4) Find the equation of the plane through $(1,2,3)$ and perpendicular to $[0,-1,1]'$?.

11. Evaluate the following integrals:

1. $\displaystyle\int \frac{x+1}{(x^2+2x+10)^4}dx$ 2. $\displaystyle\int xe^{-3x}dx$ 3. $\displaystyle\int x\sin(3x)dx$

4. $\displaystyle\int \sin x\sqrt{\cos x}dx$ 5. $\displaystyle\int e^{ax}\sin(bx)dx$ 6 $\displaystyle\int e^{ax}\cos(bx)dx$

7. $\displaystyle\int \frac{\ln x}{x^2}dx$ 8. $\displaystyle\int \frac{\ln x}{x}dx$ 9. $\displaystyle\int \frac{2x-6}{x^2+5x+13}dx$

10. $\displaystyle\int \frac{x^2+10x+12}{x^3+8x^2+12x}dx$ 11. $\displaystyle\int \frac{x^2}{x^4-1}dx$ 12. $\displaystyle\int \frac{x^3-1}{x^3+x}dx$

13. $\displaystyle\int \frac{-2x^3-x+1}{x^2}dx$ 14. $\displaystyle\frac{-2x^3-x+1}{\sqrt{x}}dx$ 15. $\displaystyle\int -4\sec^2(\frac{x}{2})-3\cos(2x)-4\sin(\frac{x}{3})dx$

16. $\displaystyle\int xe^{x^2}dx$ 17. $\displaystyle\int x^2\ln x dx$ 18. $\displaystyle\int \frac{\ln x}{x}dx$

19. $\displaystyle\int \sqrt{x}\sin(\sqrt{x})dx$ 20. $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}dx$

12. Evaluate the following limits
(a)

$$\lim_{||P||\to 0}\sum_{k=1}^{n}(c_k^2-1)\Delta x_k$$

where $P=\{x_0=1,x_1,\cdots,x_k,\cdots,x_n=2\}$, $c_k\in[x_{k-1},x_k]$, $\Delta x_k = x_k-x_{k-1}$.

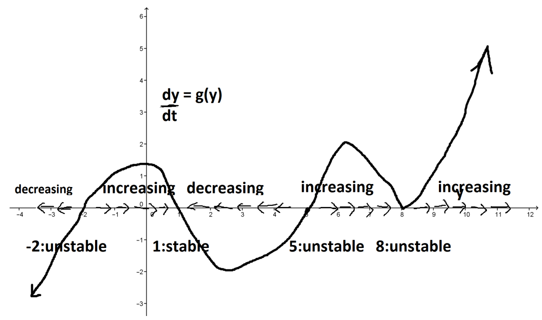
(b)

$$\lim_{||P||\to 0}\sum_{k=1}^{n}3\sin(2c_k)\Delta x_k$$

where $P=\{x_0=0,x_1,\cdots,x_k,\cdots,x_n=\frac{\pi}{4}\}$, $c_k\in[x_{k-1},x_k]$, $\Delta x_k = x_k-x_{k-1}$.

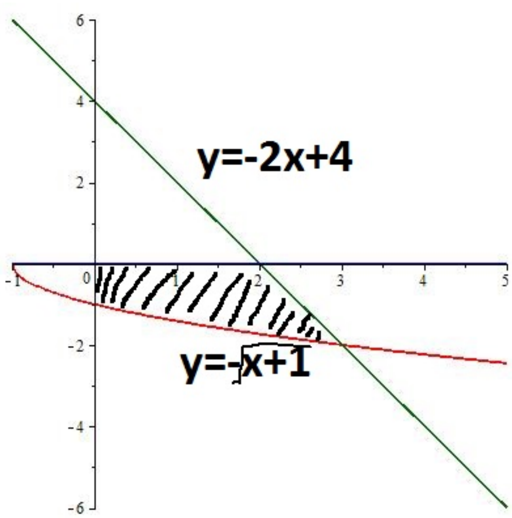
13. Solve the differential equation $\frac{dy}{dx} = y^2 - 4y + 3$ with $y(0) = 3$.

14. Suppose that $\frac{dy}{dx} = g(y)$ and the graph of $\frac{dy}{dt}$ as a function of y is given by the figure above



(a) Determine the equilibria of this differential equation.
(b) Use the graph to discuss the stability of the equilibria.
(c) What can you say about $\lim_{t\to\infty} y(t)$ if $y(0) = 3$ or $y(0) = 6$?

15. Express the area of the region enclosed by $y = -\sqrt{x+1}$, $y = -2x + 4$, $x-$axis and $y-$axis as an definite integral(**Do not evaluate the definite integral**).



Appendix B Table of the Standard Normal Distribution

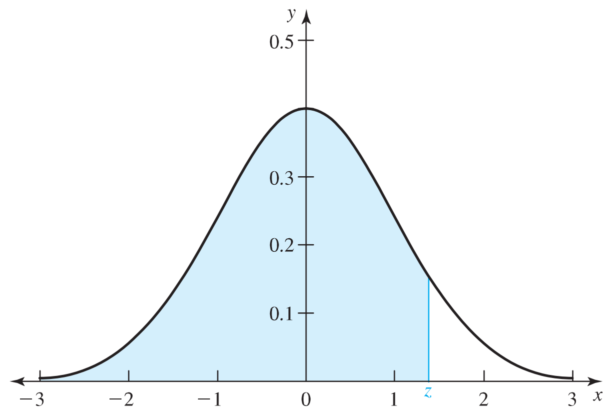


Figure B.1 Areas under the standard normal curve from $-\infty$ to z.

z	0	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5754
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7258	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7518	.7549
0.7	.7580	.7612	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7996	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986