Solution to HW 10

(1) (12.2 Problem 1) Determine the sample space for the random experiment of tossing a coin three times.
Solution: The sample space $\Omega = \{HHH, HTT, THT, TTH, THH, HTH, HHT, TTT\}$.

(2) (12. 2 Problem 5-6) Assume that $\Omega = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 3, 5\}$ and $B = \{1, 2, 3\}$

(a) Find $A \cup B$ and $A \cap B$.
Solution: $A \cup B = \{1, 2, 3, 5\}$ and $A \cap B = \{1, 3\}$.

(b) Find A^c and show that $(A^c)^c = A$.
Solution: $A^c = \Omega \backslash A = \{2, 4, 6\}$. $(A^c)^c = \Omega \backslash A^c = \{1, 3, 5\}$ So $(A^c)^c = A$.

(c) Find $(A \cup B)^c$
Solution: We have $(A \cup B)^c = (\{1, 2, 3, 5\})^c = \{4, 6\}$.

(d) Are A and B disjoint? Solution $A \cap B = \{1, 3\} \neq \emptyset$. So A and B are not disjoint.

(3) (12.2 Problem 18) Assume that $P(A) = 0.4$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$. Find $P(A \cap B)$ and $P(A^c \cap B^c)$. (Hint: Use $P(A \cup B) = P(A) + P(B) - p(A \cap B)$, $(A \cup B)^c = A^c \cap B^c$ and $P(D^c) = 1 - P(D)$.)
Solution: Using $P(A \cap B) = P(A) + P(B) - p(A \cap B)$, we have $0.7 = 0.4 + 0.4 - p(A \cap B)$ and $p(A \cap B) = 0.8 - 0.7 = 0.1$. $P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - 0.7 = 0.3$.