Quiz 2 Friday, September 6

1 Suppose the value of the function f is shown in the following table

x	1	2	3	4
f(x)	2	-1	-2	0

Approximate $\int_1^4 f(x)dx$ using 3 equal subintervals and left endpoints.

Solution.

First, we divide the interval $[1, 4]$ into 3 subintervals. The length of each interval is $\frac{end\ point-starting\ point}{the\ number\ of\ subintervals} = \frac{4-1}{3} = \frac{3}{3} = 1$.

So the first interval is $[1, 1+1] = [1, 2]$, the width of the approximate rectangle is 1, the value of f at 1 is $f(1) = 2$

The second interval is $[2, 2+1] = [2, 3]$, the width of the approximate rectangle is 1, the value of f at 2 is $f(2) = -1$

The third interval is $[3, 3+1] = [3, 4]$, the width of the approximate rectangle is 1, the value of f at 3 is $f(3) = -2$.

So the Riemann sum is

$$\underbrace{f(1)}_{height} \cdot \underbrace{1}_{width} + \underbrace{f(2)}_{height} \cdot \underbrace{1}_{width} + \underbrace{f(3)}_{height} \cdot \underbrace{1}_{width} = 2 \cdot 1 + (-1) \cdot 1 + (-2) \cdot 1 = 2 - 1 - 2 = -1.$$

Another example:Suppose the value of the function f is shown in the following table

x	1	4/3	5/3	2	7/3	8/3	3
f(x)	1	-1	2	-2	3	0	-1

Approximate $\int_1^2 f(x)dx$ using 3 equal subintervals and left endpoints.

Solution.

First, we divide the interval $[1, 2]$ into 3 subintervals. The length of each interval is $\frac{end\ point-starting\ point}{the\ number\ of\ subintervals} = \frac{2-1}{3} = \frac{1}{3}$.

So the first interval is $[1, 1+\frac{1}{3}] = [1, \frac{4}{3}]$, the width of the approximate rectangle is $\frac{1}{3}$, the value of f at 1 is $f(1) = 1$

The second interval is $[\frac{4}{3}, \frac{4}{3}+\frac{1}{3}] = [\frac{4}{3}, \frac{5}{3}]$, the width of the approximate rectangle is $\frac{1}{3}$, the value of f at $\frac{4}{3}$ is $f(\frac{4}{3}) = -1$

The third interval is $[\frac{5}{3}, \frac{5}{3}+\frac{1}{3}] = [\frac{5}{3}, 2]$, the width of the approximate rectangle is $\frac{1}{3}$, the value of f at $\frac{5}{3}$ is $f(\frac{5}{3}) = 2$.

So the Riemann sum is

$$\underbrace{f(1)}_{height} \cdot \underbrace{\frac{1}{3}}_{width} + \underbrace{f(\frac{4}{3})}_{height} \cdot \underbrace{\frac{1}{3}}_{width} + \underbrace{f(\frac{5}{3})}_{height} \cdot \underbrace{\frac{1}{3}}_{width} = 1 \cdot \frac{1}{3} + (-1) \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} = \frac{1}{3} - \frac{1}{3} + \frac{2}{3} = \frac{2}{3}.$$

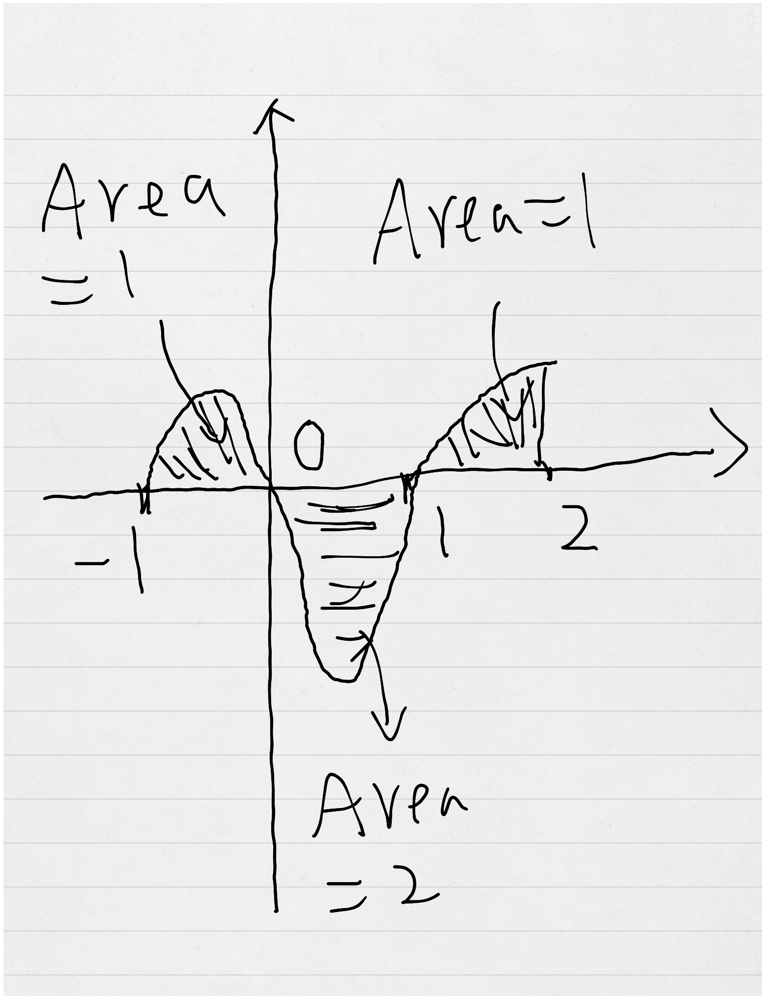

FIGURE 1.

2 Suppose the $y = f(x)$ and the area of the shaded region is given by below. Find $\int_{-1}^{1} f(x)dx$ and $\int_{-1}^{2} f(x)dx$. Solution: From the graph, we know that $\int_{-1}^{0} f(x)dx = 1$, $\int_{0}^{1} f(x)dx = -2$ and $\int_{1}^{2} f(x)dx = 1$. So we have $\int_{-1}^{1} f(x)dx = \int_{-1}^{0} f(x)dx + \int_{0}^{1} f(x)dx = 1 + (-2) = -1$ and
$\int_{-1}^{2} f(x)dx = \int_{-1}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx = 1 + (-2) + 1 = 0$.