Problem Set #1

Due: Wednesday, January 18

(1) Show that the quotient topology is indeed a topology.

(2) Show that a continuous surjective map $\pi : X \mapsto Y$ is a quotient map if and only if it takes saturated open sets to open sets, or saturated closed sets to closed sets.

(3) Show that a continuous surjective map $\pi : X \mapsto Y$ is a quotient map if X is Hausdroff and compact.

(4) Let X be the subset $\mathbb{R} \times \{0\} \cup \mathbb{R} \times \{1\}$ of $\mathbb{R}^2$. Define a equivalent relation by declaring $(x, 0) \sim (x, 1)$ if $x \neq 0$. Show that the quotient space $X \sim$ is not Hausdroff.

(5) Find a example of a quotient map $\pi : X \mapsto Y$ and a open subset $U \in X$ such that $\pi|_U$ is surjective but not a quotient map. (This is basically hw 3.9 on p62.)

Corrections to Introduction to Topological Manifolds

Ch 3 (a) **Page 52, first paragraph after Exercise 3.8:** In the first sentence, replace the words "surjective and continuous" by "surjective." Also, add the following sentence at the end of the paragraph: "It is immediate from the definition that every quotient map is continuous."

(b) **Page 52, last paragraph:** Change the word "quotient" to "surjective" in the first sentence of the paragraph.

(c) **Page 53, line 1:** Change the word "quotient" to "surjective" at the top of the page.

(d) **Page 53, Lemma 3.17:** Add the following sentence at the end of the statement of the lemma: (*More precisely, if $U \subset X$ is a saturated open or closed set, then $\pi|_U : U \to \pi(U)$ is a quotient map.*)

(e) **Page 57, second line after the first displayed diagram:** Replace the phrase "Y with the given topology is homeomorphic to Y with the quotient topology" with "the identity map is a homeomorphism between Y with the given topology and Y with the quotient topology."

(f) **Page 62, Problem 3-1:** The second part of the problem statement is false. Change the problem to the following: "Show that a finite product of open maps is open; give a counterexample to show that a finite product of closed maps need not be closed."

(g) **Page 62, Problem 3-4:** Add: "[Hint: For the unit ball in $\mathbb{R}^n$, consider the maps $\pi_i \circ \sigma^{-1} : \mathbb{R}^n \to \mathbb{R}^n$ for $1 \leq i \leq n$, where σ is stereographic projection and π_i is the projection from $\mathbb{R}^{n+1}$ to $\mathbb{R}^n$ that omits the ith coordinate.]"

(h) **Page 62, Problem 3-6:** Insert "nonempty" before "topological spaces."

Ch 7 (a) **Page 140, line 14:** Change "Step 3" to "Step 2."

(b) **Page 149, Example 7.3:** The first line should read "Define maps $f, g\colon \mathbb{R} \to \mathbb{R}^2$ by ...."

(c) **Page 156, Figure 7.7:** The labels $I \times I$, F, and X should all be in math italics.

(d) **Page 156, Exercise 7.2:** Change the first sentence to "Let X be a path connected topological space."

(e) **Page 159, second line from bottom:** "induced homeomorphism" should read "induced homomorphism."

(f) **Page 160, Proposition 7.18:** In the statement and proof of the proposition, change $(\iota_A)*$ to $(\iota_A)_*$ three times (the asterisk should be a subscript).

(g) **Page 174, proof of Lemma 7.35:** Change the word "maps" to "morphisms" (twice). Also, in the second-to-last line of the proof, change "Theorem 3.10" to "Theorem 3.11." (Actually, the last sentence is misleading, because the proof is not really exactly like that of Theorem 3.11. It would be clearer to replace the last sentence of the proof by the following: "If we take $W = P$ and $f_\alpha = \pi_\alpha$ in the diagram above, then the diagram commutes with either $f' \circ f$ or Id_P in place of f. By the uniqueness part of the defining property of the product, it follows that $f' \circ f = \mathrm{Id}_P$. A similar argument shows that $f \circ f' = \mathrm{Id}_{P'}$.")

(h) **Page 176, Problem 7-5:** Change "compact surface" to "connected compact surface."

(i) **Page 177, line 3:** The formula should read $\iota_\beta\colon X_\beta \hookrightarrow \coprod_\alpha X_\alpha$.