1. Let \( D^2 = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 \leq 1\} \). Prove that the closed unit disk \( D^2 \) in \( \mathbb{R}^2 \) cannot be retracted to the unit circle \( S^1 \).

2. Use previous problem to deduce that any continuous map \( f : D \rightarrow D \) has a fixed point. \((Hint: Prove it by contradiction. Consider the line joining \( x \) to \( f(x) \) where \( x \in D \).)\)

3. (a) The space \( G \) is a topological group meaning that \( G \) is a group and also a Hausdorff topological space such that the multiplication and map taking each element to its inverse are continuous operations. Given two loops based at the identity \( e \) in \( G \), say \( \alpha(s) \) and \( \beta(s) \), we have two ways to combine them: \( \alpha \cdot \beta \) (product of loops as in the definition of fundamental group) and secondly \( \alpha \beta \) using the group multiplication. Show, however, that these constructions give homotopic loops.

   (b) Show that the fundamental group of a path-connected topological group is abelian. \((Hint: Show that \( \alpha \beta \sim \beta \alpha \). You may want to use the hint at problem 8-3 on page 191 of the textbook.)\)

4. Show that the following are equivalent:
   (a) \( X \) is contractible.
   (b) \( X \) is homotopy equivalent to a one-point space.
   (c) Any point of \( X \) is a deformation retract of \( X \).

5. Determine the fundamental group of the Möbius band.