Problem Set #7
Due: Wednesday, Mar. 15

1. If $M$ is a connected manifold of dimension at least 3 and $q \in M$, show that
   \[ \pi_1(M \setminus \{q\}) = \pi_1(M) \] 
2. If $M$ and $M'$ are connected $n$-manifolds, $n \geq 3$, prove that
   \[ \pi_1(M_1 \# M_2) \simeq \pi_1(M_1) \ast \pi_1(M_2) \] 
3. Let $X$ be the union of the unit sphere in 3-space with the unit disk in the $x-y$ plane, i.e.
   \[ X = \{(x, y, z) | x^2 + y^2 + z^2 = 1 \text{ or } x^2 + y^2 \leq 1\} \]
   Find $\pi_1(X)$.
4. What is the fundamental group of a closed disk with two points removed?
5. What is the fundamental group of the torus with one point removed?