Problem Set #7
Due: Wednesday, Mar. 15

1. If M is a connected manifold of dimension at least 3 and $q \in M$, show that $\pi_1(M \backslash \{q\}) = \pi_1(M)$.

2. If M and M are connected n-manifolds, $n \geq 3$, prove that
$$\pi_1(M_1 \sharp M_2) \simeq \pi_1(M_1) * \pi_1(M_2).$$

3. Let X be the union of the unit sphere in 3-space with the unit disk in the $x - y$ plane, i.e. $X = \{(x, y, z) | x^2 + y^2 + z^2 = 1 \text{ or } x^2 + y^2 \leq 1\}$. Find $\pi_1(X)$.

4. What is the fundamental group of a closed disk with two points removed?

5. What is the fundamental group of the torus with one point removed?