Problem Set #8
Due: Wednesday, Apr. 12

1. Prove that if $X$ is compact and $f : X \rightarrow Y$ is a local homeomorphism, then, for any point $y \in Y$, $f^{-1}(y)$ is a finite set. If it is also assumed that $Y$ is a connected Hausdorff space, then $f$ maps $X$ onto $Y$.

2. Assume $X$ and $Y$ are path connected and locally path connected, $X$ is compact Hausdorff, and $Y$ is Hausdorff. Let $f : X \rightarrow Y$ be a local homeomorphism; prove that $(X, f)$ is a covering space of $Y$. (warning: This exercise is more subtle than it looks!)

3. Suppose $p : \tilde{X} \rightarrow X$ is a covering map and $X$ is a compact manifold. Show that $\tilde{X}$ is compact if and only if $p$ is a finite-sheeted covering.

4. Show that there is a two-sheeted covering of the Klein bottle by the torus.