Problem Set #8

Due: Wednesday, Apr. 12

1. Prove that if X is compact and $f : X \mapsto Y$ is a local homeomorphism, then, for any point $y \in Y$, $f^{-1}(y)$ is a finite set. If it is also assumed that Y is a connected Hausdorff space, then f maps X onto Y.

2. Assume X and Y are path connected and locally path connected, X is compact Hausdorff, and Y is Hausdorff. Let $f : X \mapsto Y$ be a local homeomorphism; prove that (X, f) is a covering space of Y. (warning: This exercise is more subtle than it looks!)

3. Suppose $p : \widetilde{X} \mapsto X$ is a covering map and X is a compact manifold. Show that $\widetilde{X}$ is compact if and only if p is a finite-sheeted covering.

4. Show that there is a two-sheeted covering of the Klein bottle by the torus.