Calculus III Chapter 14

Thomas 12th edition

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What we have done so far?

We covered space curves, introduced notions of velocity, acceleration, arc-length, unit tangent vectors, and we calculated them. Then we integrated differential equations of first and second order and applied that to the case of projectile motion. I planned to cover curvature and torsion and Kepler's Laws but due to unforeseen glitches with computers and my hand I lost time but I plan to recover and go back to them sometime later! I plan to post my lecture notes on my web-site very soon. Thank you for your patience.

Examples of Functions of Several Variables

Local temperature T varies from place to place and each place is uniquely identified by its latitude ϕ and longitude θ. Thus

$$T(\phi, \theta)$$

is function of two variables. We can make it into a function of 3 variables if we include altitude and think of all the space above our planet. Similarly barometric pressure is also function of two or 3 variables depending on your considerations.

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are a few I wrote down but you can dream up any number of these on your own. In order to get an intimate grasp of these functions we introduce the notion of level sets of a function, in the case of two variables these level sets are in general curves and the case of three variables they are level surfaces.

Level sets and graphing

For example level sets of $x^2 + y^2$ are circles and level sets of $x + y + z$ are planes. We can use Maple software to plot these curves and surfaces.

Continuity

The list of functions above are either polynomials or compositions of polynomials with sin cos functions. Clearly they are continuous by using the Theorem 1 on page 775. But occasionally one may come across a point where the Theorem will not apply, take for example

$$f(x,y) = \frac{xy^2}{x^2+y^2}.$$

Theorem 1 will guarantee continuity at all points except at the origin $(0,0)$. When it is not at the origin both numerator and denominator are continuous and the denominator does not vanish and the Theorem applies. But at the origin both numerator and denominator vanish and the Theorem does not apply. But notice the inequality

$$|xy| \leq x^2 + y^2,$$

and hence

$$|xy^2| \leq |y|(x^2+y^2).$$

If $x^2 + y^2 \neq 0$, we can divide and get

$$|f(x, y)| \leq |y|,$$

and since $|y|$ tends to zero as (x, y) tend to the origin, we obtain that the limit of f is zero at the origin. By setting $f(0, 0) = 0$, we can make f to be continuous everywhere.But the function $g(x, y) = \frac{xy}{x^2+y^2}$ is not that lucky. As we approach zero it takes infinitely many limits! Check it out!

Limits of Quotients

Here is a list of interesting problems in computing limits from page 782 as (x, y) goes to the origin.

1. $\frac{x^3 - xy^2}{x^2 + y^2}$
2. $\cos\left(\frac{x^3 - y^3}{x^2 + y^2}\right)$
3. $\frac{2x}{x^2 + x + y^2}$

Differentiation