## **OMT019 - ELEMENTARY LINEAR ALGEBRA**

## **3-4 Semester Hours/4-5 Quarter Hours**

Related TAGs: Math, Physics

In an Elementary Linear Algebra course, students should:

- develop mathematical thinking and communication skills and learn to apply precise, logical reasoning to problem solving (e.g., teaching computational techniques should not override the goal of leading students to understand fundamental mathematical relationships).
- be able to communicate the breadth and interconnections of the mathematical sciences through being presented key ideas and concepts from a variety of perspectives, a broad range of examples and applications, connections to other subjects, and contemporary topics and their applications.
- experience geometric as well as algebraic viewpoints and approximate as well as exact solutions.
- use computer technology to support problem solving and to promote understanding (e.g., linear algebra courses can use technology for matrix manipulation or for visualizing the effects of linear transformations in two or three dimensions, and technology makes large linear systems tractable).
- for students in the mathematical sciences, progress from a procedural/computational understanding of mathematics to a broad understanding encompassing logical reasoning, generalization, abstraction, and formal proof; gain experience in careful analysis of data; and become skilled at conveying their mathematical knowledge in a variety of settings, both orally and in writing.
- Adapted from the MAA/CUPM 2004 Curriculum Guide

The prerequisite for an Elementary Linear Algebra course is generally either a year-long sequence in calculus (TMM005 Calculus I & TMM006 Calculus II) or OMT018 Calculus III. Adapted statements regarding Calculus in the MAA/CUPM 2004 Curriculum Guide also generally apply to Elementary Linear Algebra.

In addition, any elementary linear algebra course should

- respond to the needs of client disciplines;
- focus on a being a matrix-oriented course;
- maintain a strong geometric emphasis;
- consider the needs and interests of students as learners;
- utilize technology to reinforce concepts, contribute to discoveries, and support finding solutions to realistic applied problems; and
- lead to a second course in matrix theory/linear algebra.
- Adapted from the 1993 Linear Algebra Curriculum Study Group Recommendations

To qualify for TAG equivalency of OMT019 (Elementary Linear Algebra), a course must cover as a minimum the essential learning outcomes, denoted by an asterisk (\*). An Elementary Linear Algebra course may also commonly include some of the listed nonessential learning outcomes. These optional topics should be included only if there is adequate course time to do so beyond giving primary course attention to the essential learning outcomes. At least 70% of the classroom instructional time has to be spent on the essential learning outcomes. The optional learning outcomes are learning experiences that enhance, reinforce, enrich or are further applications of the essential learning outcomes. If review of prerequisite course content is necessary, only a minimal amount of time should be devoted to such review .

The successful Linear Algebra student should be able to:

- 1. Understand algebraic and geometric representations of vectors in R nand their operations, including addition, scalar multiplication and dot product. understand how to determine the angle between vectors and the orthogonality of vectors.\*
- 2. Solve systems of linear equations using Gauss-Jordan elimination to reduce to echelon form. Solve systems of linear equations using the inverse of the coefficient matrix when possible. Interpret existence and uniqueness of solutions geometrically.\*
- 3. Perform common matrix operations such as addition, scalar multiplication, multiplication, and transposition. Discuss associativity and noncommutativity of matrix multiplication.\*
- 4. Discuss spanning sets and linear independence for vectors in R n.

For a subspace of R n, prove all bases have the same number of elements and define the dimension. Prove elementary theorems concerning rank of a matrix and the relationship between rank and nullity.\*

- 5. Interpret a matrix as a linear transformation from R *n* to R *m*. Discuss the transformation's kernel and image in terms of nullity and rank of the matrix. Understand the relationship between a linear transformation and its matrix representation, and explore some geometric transformations in the plane. Interpret a matrix product as a composition of linear transformations.\*
- 6. Use determinants and their interpretation as volumes. Describe how row operations affect the determinant. Analyze the determinant of a product algebraically and geometrically.\*
- 7. Define eigenvalues and eigenvectors geometrically. Use characteristic polynomials to compute eigenvalues and eigenvectors. Use eigenspaces of matrices, when possible, to diagonalize a matrix.\*
- 8. Use axioms for abstract vector spaces (over the real or complex fields) to discuss examples (and non-examples) of abstract vector spaces such as subspaces of the space of all polynomials.\*
- 9. Discuss the existence of a basis of an abstract vector space. Describe coordinates of a vector relative to a given basis. For a linear transformation between vector spaces, discuss its matrix relative to given bases. Discuss how those matrices changes when the bases are changed.
- 10. Discuss orthogonal and orthonormal bases, Gram-Schmidt orthogonalization, orthogonal complements and projections. Discuss rigid motions and orthogonal matrices.
- 11. Discuss general inner product spaces and symmetric matrices, and associated norms. Explain how orthogonal projections relate to least square approximations.