1. Give the definition of continuity for a function from one topological space to another. Use your definition to show that $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \frac{1}{x^2}$ is not continuous.

2. Prove that the space $(-1, 1)$ is not homeomorphic to $[-1, 1]$.

3. Prove that a closed subset of a compact topological space is compact.

4. Prove that in a Hausdorff topological space a compact subset is closed.

5. Prove that in a metric space a compact subset is closed and bounded.

6. Prove that a closed and surjective map is a quotient map.

7. Prove that for any given subset $E$ in a metric space $X$, the function $f : X \rightarrow \mathbb{R}$ defined by $f(x) = d(x, E)$, \((x \in X)\), is continuous.

8. Let $A$ be a subspace of the topological space $X$. Define what it means for a subset $B$ of $A$ to be closed in $A$ in the subspace topology. Prove carefully from your definition that $B$ is closed in $A$ if and only if $B$ is closed in $X$.

9. Define compactness for a topological space. A collection of subsets is said to have the finite intersection property if every finite class of those subsets has non-empty intersection. Prove that a topological space is compact iff every collection of closed subsets that has the finite intersection property itself has non-empty intersection.

10. Let $f : X \rightarrow Y$ be a quotient map, with $Y$ connected. Show that if $f^{-1}(y)$ is connected for all $y \in Y$, then $X$ is connected.

11. Prove that the intersection of two open dense subsets of a topological space is dense.

12. Let $X$ be a topological space. Let $A \subset X$ be connected. Prove that the closure $\bar{A}$ of $A$ is connected.