Do no more than five (5) questions. If you think there is a misprint in a question state your query clearly and try to interpret it in a non-trivial manner.

The Examination Committee tries to proofread the exams as carefully as possible. Nevertheless, the exam may contain misprints. If you are convinced a problem has been stated incorrectly, mention this to the proctor and indicate your interpretation in your solution. In such cases do not interpret the problem in such a way that it becomes trivial.
Exam is two hours. Do five (5) problems.

1. Define what it means for a topological space to be disconnected. Prove that a space is disconnected if and only if there is a continuous map from the space onto the discrete two point space \{0,1\}.

2. State whether the following propositions are true or false. If they are true prove them. If they are false give a counterexample.

   (a) In a compact topological space a closed subspace is compact.
   (b) In a metric space a compact subspace is closed.
   (c) In a Hausdorff space a compact subspace is closed.

3. Prove that a topological space $X$ is Hausdorff if and only if the diagonal $\Delta = \{(x,x) \in X \times X : x \in X\}$ is closed in $X \times X$ where $X \times X$ has the product topology induced by $X$.

4. Prove that a continuous bijection from a compact topological space onto a Hausdorff topological space is necessarily a homeomorphism.

5. Define the product topology $X \times Y$ on topological spaces $X,Y$. Prove from your definition that each of the projections from $X \times Y$ to $X$ and $Y$, respectively, are both continuous and open maps.

6. Define what it means for $p : X \rightarrow Y$ to be a quotient map of topological spaces or, equivalently, for $p$ to be an identification map. Verify in detail that $\exp : \mathbb{R} \rightarrow \mathbb{R}^2$ given by $\exp(t) = (\cos t, \sin t)$ gives an identification map of $\mathbb{R}$ onto the unit circle.

7. Prove that as subspaces of $\mathbb{R}$ with the usual topology the open interval $(0,1)$ is not homeomorphic to the half-open interval $[0,1)$.