Math 2890 – Exam 1
Summer IV – 2009
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Instructions: Show your work! Explain your answers. No books. No notes. Non-graphing calculators only. Please write neatly.

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1. Write the augmented matrix that corresponds to the linear system below.

$$\begin{array}{rcrcrcrcrcr} -6x_1 & - & 4x_2 & - & x_3 & + & x_4 & + & 4x_5 & = & 5 \\ 5x_1 & + & 6x_2 & + & 2x_3 & + & x_4 & + & 4x_5 & = & 6 \\ -5x_1 & & & - & 6x_3 & - & 6x_4 & + & 5x_5 & = & -5 \\ & & 5x_2 & - & 5x_3 & - & 3x_4 & + & 3x_5 & = & -2 \end{array}$$

answer:

$$\left(\begin{array}{rrrrr|r} -6 & -4 & -1 & 1 & 4 & 5 \\ 5 & 6 & 2 & 1 & 4 & 6 \\ -5 & 0 & -6 & -6 & 5 & -5 \\ 0 & 5 & -5 & -3 & 3 & -2 \end{array}\right)$$

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2. Write the linear system that corresponds to the augmented matrix below.

$$\left(\begin{array}{rrrr|r} 0 & -2 & -4 & -4 & -2 \\ 5 & -2 & -6 & 6 & 6 \\ 1 & 4 & 1 & -6 & 5 \\ 2 & 3 & -5 & 4 & -4 \\ 5 & -3 & 0 & 5 & -1 \end{array}\right)$$

answer:

$$\begin{array}{rcrcrcrcr} & & -2x_2 & - & 4x_3 & - & 4x_4 & = & -2 \\ 5x_1 & - & 2x_2 & - & 6x_3 & + & 6x_4 & = & 6 \\ x_1 & + & 4x_2 & + & x_3 & - & 6x_4 & = & 5 \\ 2x_1 & + & 3x_2 & - & 5x_3 & + & x_4 & = & -4 \\ 5x_1 & - & 3x_2 & & & + & 5x_4 & = & -1 \end{array}$$

3. Let $\alpha = -5$, $\beta = 2$, $v = \begin{pmatrix} 6 \\ -1 \\ 0 \\ 1 \end{pmatrix}$ and $w = \begin{pmatrix} -7 \\ 1 \\ -2 \\ -2 \end{pmatrix}$.

Compute the linear combination $v\alpha + w\beta$.

answer: The linear combination $v\alpha + w\beta = \begin{pmatrix} -44 \\ 7 \\ -4 \\ -9 \end{pmatrix}$.

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4. Let $A = \begin{pmatrix} 7 & -1 & 7 & 2 \\ 2 & 5 & 3 & 3 \\ 9 & 3 & -8 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -9 & 5 & -6 & 6 \\ -2 & 6 & 7 & 3 \\ 8 & 9 & 6 & -2 \end{pmatrix}$.

Compute the sum $A + B$ if it is defined; otherwise, explain why it is not defined.

answer: The sum $A + B = \begin{pmatrix} -2 & 4 & 1 & 8 \\ 0 & 11 & 10 & 6 \\ 17 & 12 & -2 & -4 \end{pmatrix}$.

5. Let $A = \begin{pmatrix} -2 & 8 & 6 & -6 \\ -4 & 20 & -4 & 0 \\ 8 & -32 & -22 & 34 \end{pmatrix}$.

Use Gaussian elimination to reduce the matrix A to row echelon form.

answer: Gaussian elimination reduces A to $\begin{pmatrix} -2 & 8 & 6 & -6 \\ 0 & 4 & -16 & 12 \\ 0 & 0 & 2 & 10 \end{pmatrix}$.

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6. Let $A = \begin{pmatrix} -2 & 8 & 6 & -6 \\ -4 & 20 & -4 & 0 \\ 8 & -32 & -22 & 34 \end{pmatrix}$.

Use Gaussian elimination with partial pivoting to reduce the matrix A to row echelon form.

answer: Gaussian elimination with partial pivoting reduces A to $\begin{pmatrix} 8 & -32 & -22 & 34 \\ 0 & 4 & -15 & 17 \\ 0 & 0 & 0.5 & 2.5 \end{pmatrix}$.

7. Let $A = \begin{pmatrix} 0 & -2 & -4 & 8 & -2 & -2 \\ 5 & 5 & 30 & -20 & 4 & 28 \\ 4 & 1 & 18 & -4 & 1 & 21 \end{pmatrix}$ and $b = \begin{pmatrix} -4 \\ -31 \\ -34 \end{pmatrix}$.

Find the general solution of the equation $Ax = b$. HINT: The augmented system $(A|b)$ has reduced row echelon form $\left(\begin{array}{cccccc|c} 1 & 0 & 4 & 0 & 0 & 5 & -9 \\ 0 & 1 & 2 & -4 & 0 & -1 & 6 \\ 0 & 0 & 0 & 0 & 1 & 2 & -4 \end{array}\right)$.

answer: The general solution is $x = \begin{pmatrix} -9 \\ 6 \\ 0 \\ 0 \\ -4 \\ 0 \end{pmatrix} + \begin{pmatrix} -4 & 0 & -5 \\ -2 & 4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 1 \end{pmatrix} y$, where $y \in R^3$.

8. Let $u = \begin{pmatrix} -1 \\ 1 \\ -5 \end{pmatrix}$, $v = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$, $w = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$ and $x = \begin{pmatrix} -2 \\ 2 \\ -10 \end{pmatrix}$.

   Do the given vectors span $\mathbb{R}^3$?

   answer: The vectors span $\mathbb{R}^3$ since (after constructing a matrix using the vectors as the columns) every row has a pivot.

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9. Let $u = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 7 \\ 1 \end{pmatrix}$ and $w = \begin{pmatrix} 1 \\ 11 \\ 8 \end{pmatrix}$.

   Are the given vectors linearly independent?

   answer: The vectors are linearly dependent since (after constructing a matrix using the vectors as the columns) there is no pivot in column 3.

10. Let $A = \begin{pmatrix} 7 & -2 & 3 \\ -2 & -7 & -2 \\ 7 & 8 & 1 \\ 2 & -2 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 7 & -1 \\ 4 & -2 & 5 \\ -4 & 9 & 1 \end{pmatrix}$.

Compute column 2 of AB by first writing it as a linear combination of the columns of A.

answer: $\begin{pmatrix} 7 \\ -2 \\ 7 \\ 2 \end{pmatrix}(7) + \begin{pmatrix} -2 \\ -7 \\ 8 \\ -2 \end{pmatrix}(-2) + \begin{pmatrix} 3 \\ -2 \\ 1 \\ -2 \end{pmatrix}(9) = \begin{pmatrix} 80 \\ -18 \\ 42 \\ 0 \end{pmatrix}$

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11. Let $A = \begin{pmatrix} -6 & -8 & -1 \\ -9 & -9 & -5 \\ 2 & -4 & 3 \\ 7 & 8 & -9 \\ 2 & -4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -8 & -9 & 5 & 5 \\ -8 & 2 & -1 & 3 \\ -6 & 7 & 1 & -6 \end{pmatrix}$.

Compute row 3 of AB by first writing it as a linear combination of the rows of B.

answer: $(2)\begin{pmatrix} -8 & -9 & 5 & 5 \end{pmatrix} + (-4)\begin{pmatrix} -8 & 2 & -1 & 3 \end{pmatrix} + (3)\begin{pmatrix} -6 & 7 & 1 & -6 \end{pmatrix} = \begin{pmatrix} -2 & -5 & 17 & -20 \end{pmatrix}$

12. Let $A = \begin{pmatrix} -1 & 9 & -3 & -6 \\ 4 & 2 & 4 & 0 \\ -5 & -9 & 4 & -2 \\ 7 & -4 & -3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 5 & 5 & -8 \\ 7 & -3 & -6 & 4 \\ 5 & -2 & 4 & -3 \\ -3 & 4 & -5 & -1 \end{pmatrix}$.

Compute the entry of AB in row 2, column 4.

answer: $\begin{pmatrix} 4 & 2 & 4 & 0 \end{pmatrix} \begin{pmatrix} -8 \\ 4 \\ -3 \\ -1 \end{pmatrix} = -36.$

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13. Let $A = \begin{pmatrix} -4 & -6 & -7 \\ 0 & -1 & 6 \\ 4 & 5 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & 9 \\ -2 & 5 \\ 8 & 8 \end{pmatrix}$.

Compute the product AB as a sum of 3 rank one matrices.

answer: $AB = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} \begin{pmatrix} -5 & 9 \end{pmatrix} + \begin{pmatrix} -6 \\ -1 \\ 5 \end{pmatrix} \begin{pmatrix} -2 & 5 \end{pmatrix} + \begin{pmatrix} -7 \\ 6 \\ 8 \end{pmatrix} \begin{pmatrix} 8 & 8 \end{pmatrix}$

$= \begin{pmatrix} 20 & -36 \\ 0 & 0 \\ -20 & 36 \end{pmatrix} + \begin{pmatrix} 12 & -30 \\ 2 & -5 \\ -10 & 25 \end{pmatrix} + \begin{pmatrix} -56 & -56 \\ 48 & 48 \\ 64 & 64 \end{pmatrix}$