Math 2890 – Exam 2
Summer IV – 2009
Odenthal

> **Instructions:** Show your work! Explain your answers. No books. No notes. Non-graphing calculators only. Please write neatly. All problems are worth 10 points except as indicated.

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1. Let $A = \begin{pmatrix} 4 & 4 & 16 & 0 \\ 2 & 2 & 8 & -2 \\ 1 & 1 & 4 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

   Find the rank of the matrix A.

   answer: The rank of A is 2

2. Let $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 6 & 1 & 0 & 0 \\ -4 & 2 & 1 & 0 \\ -5 & 0 & -9 & 1 \end{pmatrix}$.

Find the inverse of A if it exists.

answer: $A^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -6 & 1 & 0 & 0 \\ 16 & -2 & 1 & 0 \\ 149 & -18 & 9 & 1 \end{pmatrix}$.

3. Let $L = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 2 & 4 & 1 \end{pmatrix}$ $U = \begin{pmatrix} -1 & 2 & -3 \\ 0 & -2 & 5 \\ 0 & 0 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ -7 \\ -1 \end{pmatrix}$.

   Use the LU factorization $A = LU$ to solve the matrix equation $Ax = b$.

   answer: The vector $y = \begin{pmatrix} 2 \\ 1 \\ -9 \end{pmatrix}$ is the solution of the system $Ly = b$.

   and the vector $x = \begin{pmatrix} -9 \\ -8 \\ -3 \end{pmatrix}$ is the solution of $Ux = y$ and so of $Ax = b$.

4. Let $A = \begin{pmatrix} 3 & 3 & -3 \\ 9 & 14 & -11 \\ 15 & -10 & -1 \end{pmatrix}$.

Use Gaussian Elimination (or Wedderburn rank reduction) to find the LU factorization of the matrix A.

answer: $A = LU$ where $L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & -5 & 1 \end{pmatrix}$ and $U = \begin{pmatrix} 3 & 3 & -3 \\ 0 & 5 & -2 \\ 0 & 0 & 4 \end{pmatrix}$

5. Let $A = \begin{pmatrix} -1 & -8 & -5 \\ -8 & -8 & 1 \\ 9 & 2 & 0 \end{pmatrix}$.

Use Gaussian Elimination with Partial Pivoting (or Wedderburn rank reduction) to find a permuted LU factorization of the matrix A.

answer: $A = LU$ where $L = \begin{pmatrix} -0.1111 & 1 & 0 \\ -0.8889 & 0.8 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ and $U = \begin{pmatrix} 9 & 2 & 0 \\ 0 & -7.7778 & -5 \\ 0 & 0 & 5 \end{pmatrix}$

6. Let $A = \begin{pmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 5 & 4 & 3 & 0 & 0 & 0 \\ 1 & 1 & 4 & 3 & 0 & 0 \\ 4 & 4 & 4 & 2 & 2 & 0 \\ 4 & 2 & 3 & 5 & 5 & 4 \end{pmatrix} \begin{pmatrix} 2 & 2 & 4 & 1 & 5 & 5 \\ 0 & 4 & 1 & 0 & 3 & 3 \\ 0 & 0 & 3 & 0 & 1 & 3 \\ 0 & 0 & 0 & 4 & 3 & 5 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{pmatrix}$.

Find the determinant of A if it exists.

answer: The matrix A has determinant $(1440)(960) = 1382400$.

7. Let $A = \begin{pmatrix} 8 & 8 & 4 & 9 & 1 & 0 \\ 6 & 3 & 0 & 6 & 0 & 8 \\ 0 & 0 & 8 & 7 & 4 & 4 \\ 0 & 0 & 3 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 & 9 & 3 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{pmatrix}$.

Find the determinant of A if it exists.

answer: The matrix has determinant $(-24)(-5)(-6) = -720$.

8. Let $A = \begin{pmatrix} 1 & 3 & 13 \\ 1 & 4 & 16 \\ 3 & 8 & 36 \end{pmatrix}$ and $v = \begin{pmatrix} -5 \\ -9 \\ -10 \end{pmatrix}$.

(a) Is v in the column space of A?

(b) Is v in the null space of A?

answer: (a) Since $(A|v)$ has a pivot in the last column $Ax = v$ is not consistent, and v is **not** in the column space of A.

(b) v is **not** in the null space of A since $A \neq 0$.

9. Let $A = \begin{pmatrix} 1 & -4 & 17 \\ 4 & 0 & -12 \\ 4 & 5 & -37 \end{pmatrix}$

(a) Find a nonzero vector in the column space of A.

(b) Find a nonzero vector in the null space of A.

answer: (a) I found the vector $\begin{pmatrix} -24 \\ -16 \\ 9 \end{pmatrix}$ in the column space. You'll probably find a different vector.

(b) I found the vector $\begin{pmatrix} -15 \\ -25 \\ -5 \end{pmatrix}$ in the null space. You'll probably find a different vector.

10. Let

$$A = \begin{pmatrix} 1/2 & -1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 & 1/2 \end{pmatrix}$$

Is A orthogonal?

answer: Yes

11. (20 points) Let $A = \begin{pmatrix} 1322 & -3966 & 232 & 42 & 5816 & -70 \\ 500 & -1500 & 83 & 13 & 2197 & -28 \\ -107 & 321 & -18 & -3 & -470 & 6 \\ 61 & -183 & 11 & 2 & 269 & -3 \\ -81 & 243 & -15 & -3 & -357 & 4 \end{pmatrix}$.

Find bases for Col(A), Nul(A), Col(A^T) and Nul(A^T).

Hint: If you form the matrix $(A|I)$ and use row operations to put the A part in reduced row echelon form you get $(R|S)$ where

$$R = \begin{pmatrix} 1 & -3 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \ S = \begin{pmatrix} 1 & -1 & 2 & 2 & 9 \\ -5 & 6 & -5 & -12 & -47 \\ 5 & -9 & -9 & 20 & 53 \\ 5 & -5 & 11 & 13 & 46 \\ 1 & 2 & 16 & -10 & 0 \end{pmatrix}.$$

answer: The columns of the matrix $\begin{pmatrix} 1322 & 232 & 42 & -70 \\ 500 & 83 & 13 & -28 \\ -107 & -18 & -3 & 6 \\ 61 & 11 & 2 & -3 \\ -81 & -15 & -3 & 4 \end{pmatrix}$ form a basis for the column space of A.

The columns of the matrix $\begin{pmatrix} 3 & -4 \\ 1 & 0 \\ 0 & -3 \\ 0 & 4 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$ form a basis for the null space of A.

The columns of the matrix $\begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 4 & 3 & -4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ form a basis for the column space of A^T.

The columns of the matrix $\begin{pmatrix} 1 \\ 2 \\ 16 \\ -10 \\ 0 \end{pmatrix}$ form a basis for the null space of A^T.