Math 2890 – Exam 3
Summer IV – 2009
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> **Instructions:** Show your work! Explain your answers. No books. No notes. Non-graphing calculators only. Please write neatly.

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1. Let $A = \begin{pmatrix} -17 & -6 \\ 84 & 28 \end{pmatrix}$ and $\lambda = 7$.

   Find an eigenvector for the matrix A that corresponds to the given eigenvalue λ.

   answer: One choice is $x = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$.

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2. Let $A = \begin{pmatrix} -37 & 13 & 4 \\ -64 & 24 & 8 \\ -8 & -8 & -4 \end{pmatrix}$ and $x = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$.

   Find the eigenvalue for the matrix A that corresponds to the given eigenvector x.

   answer: $\lambda = -8$.

3. Let $A = \begin{pmatrix} -4 & -60 & 27 \\ 0 & -8 & 3 \\ 0 & -14 & 5 \end{pmatrix}$.

Find the eigenvalues (including multiplicities) of A.

answer: The eigenvalues are: $-1, -2, -4$.

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4. Let $A = \begin{pmatrix} 0 & 2 \\ 2 & 1 \end{pmatrix}$ and $x_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Use the Power Method to find estimates μ_3 and x_3 for the dominant eigenvalue of A and its eigenvector. Give your answer either as rational numbers or decimals with at least four digits of accuracy.

answer: $\mu_3 = 2.47368$ and $x_3 = \begin{pmatrix} 0.73684 \\ 1 \end{pmatrix}$

details: The vector $x_{k+1} = Ax_k(1/\mu_k)$ where μ_k is an entry in Ax_k whose absolute value is as large as possible.

k	0	1	2	3	4
x_k	1	0.66667	0.85714	0.73684	0.80851
	1	1.00000	1.00000	1.00000	1.00000
y_k	2	2.00000	2.00000	2.00000	2.00000
	3	2.33333	2.71429	2.47368	2.26170
μ_k	3	2.33333	2.71429	2.47368	2.26170

5. Let $A = \begin{pmatrix} 1 & 0 \\ 4 & 3 \end{pmatrix}$.

   Find an invertible matrix P and a diagonal matrix D such that $AP = PD$, or explain why no such matrices exist.

   answer: One choice is $P = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$.

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6. Let $x_1 = \begin{pmatrix} -1 \\ 8 \end{pmatrix}$ $x_2 = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$

   and $\lambda_1 = -3, \lambda_2 = -2$.

   Write down a matrix A that has the given vectors as eigenvectors with the corresponding scalars as the eigenvalues.

   answer: One solution is $A = PDP^{-1}$ where $P = \begin{pmatrix} -1 & 8 \\ 8 & -4 \end{pmatrix}$ and $D = \begin{pmatrix} -3 & 0 \\ 0 & -2 \end{pmatrix}$.

7. Let $\{y_0, y_1, y_2, \ldots\} = \{3, -1, -2, 1, -4, 3, 3, -1, -4, 3, \ldots\}$.

   Use the filter $z_k = -5y_{k+2} + 2y_{k+1} + 2y_k$ to find the first 3 terms of the signal $\{z_0, z_1, z_2, \ldots\}$.

   answer: $\{z_0, z_1, z_2, \ldots\} = \{14, -11, 18, -21, \ldots\}$

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8. Let $A = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$.

   Compute A^{10}.

   HINT: It may help to know that $AP = PD$ where

   $P = \begin{pmatrix} 1 & 3 \\ -1 & -2 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ while $P^{-1} = \begin{pmatrix} -2 & -3 \\ 1 & 1 \end{pmatrix}$.

   answer:

$$
\begin{aligned}
A^{10} &= PD^{10}P^{-1} \\
&= \begin{pmatrix} 1 & 3 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1024 \end{pmatrix} \begin{pmatrix} -2 & -3 \\ 1 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 3070 & 3069 \\ -2046 & -2045 \end{pmatrix}.
\end{aligned}
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9. Consider $y_{k+2} - 6y_{k+1} - 16y_k = -42$

   Find the general solution of this linear difference equation.

   answer: The general solution is $y_k = c_1(8)^k + c_2(-2)^k + 2$.

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10. Let $A = \begin{pmatrix} 0.9 & 0.3 \\ 0.1 & 0.7 \end{pmatrix}$.

    Find a steady state probability vector for the stochastic matrix A.

    answer: $x = \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix}$.

11. Let $A = \begin{pmatrix} -2.8 & -4.4 \\ 3.3 & 4.9 \end{pmatrix}$.

Is the origin an attractor, repellor or saddle point for the discrete dynamical system $x_{k+1} = Ax_k$?

HINT: It may help to know that $AP = PD$ where $P = \begin{pmatrix} 1 & 4 \\ -1 & -3 \end{pmatrix}$ and $D = \begin{pmatrix} 1.6 & 0 \\ 0 & 0.5 \end{pmatrix}$.

answer: The origin is a saddle point since A has eigenvalues both greater than and less than 1 in absolute value.

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12. Let $A = \begin{pmatrix} -75 & -36 \\ 162 & 78 \end{pmatrix}$.

Is the origin an attractor, repellor or saddle point for the differential equation $y' = Ay$?

HINT: It may help to know that $AP = PD$ where $P = \begin{pmatrix} 1 & -4 \\ -2 & 9 \end{pmatrix}$ and $D = \begin{pmatrix} -3 & 0 \\ 0 & 6 \end{pmatrix}$.

answer: The origin is a saddle point since A has both positive and negative eigenvalues.

13. Suppose $AP = PD$ where $P = \begin{pmatrix} 1 & 4 \\ 6 & 8 \end{pmatrix}$ and $D = \begin{pmatrix} 0.4 & 0 \\ 0 & 1.5 \end{pmatrix}$.

Solve the discrete dynamical system $x_{k+1} = Ax_k$ where $x_0 = \begin{pmatrix} 34 \\ 92 \end{pmatrix}$.

answer: $x_k = \begin{pmatrix} 1 \\ 6 \end{pmatrix} (6)(0.4)^k + \begin{pmatrix} 4 \\ 8 \end{pmatrix} (7)(1.5)^k$

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14. Suppose $AP = PD$ where $P = \begin{pmatrix} -1 & 1 \\ -5 & 2 \end{pmatrix}$ and $D = \begin{pmatrix} 8 & 0 \\ 0 & -5 \end{pmatrix}$.

Solve the initial value problem $y' = Ay$ where $y(0) = \begin{pmatrix} -8 \\ -28 \end{pmatrix}$.

answer: $y = \begin{pmatrix} -1 \\ -5 \end{pmatrix} (4)e^{8t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} (-4)e^{-5t}$

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