2 Pages **Quiz 2A**, Math 1850 Section 005
9-10-09 Solutions Name ____________________

(12) 1. Evaluate the limit, if it exists.

(a) $\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4} \left(=\frac{0}{0}\right) = \lim_{x\to 4} \frac{x(x-4)}{(x-4)(x+1)} = \lim_{x\to 4} \frac{x}{x+1} = \frac{4}{5}$

(b) $\lim_{h\to 0} \frac{(6+h)^2-36}{h} \left(=\frac{0}{0}\right) = \lim_{h\to 0} \frac{36+12h+h^2-36}{h} = \lim_{h\to 0} \frac{h(12+h)}{h}$

$$= \lim_{h\to 0} 12+h = 12$$

(c) $\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h} \left(=\frac{0}{0}\right) = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{3+h}-\frac{1}{h}\right] = \lim_{h\to 0} \frac{1}{h}\left[\frac{3-(3+h)}{(3+h)3}\right]$

$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-h)}{(3+h)3}\right] = \lim_{h\to 0}\left[\frac{-1)}{(3+h)3}\right] = -\frac{1}{9}$ by finding a common denominator.

(d) $\lim_{x\to 0} \frac{\sin 4x}{\sin 6x} \left(=\frac{0}{0}\right) = \lim_{x\to 0} \frac{\sin 4x}{x}\frac{x}{\sin 6x} = \frac{4}{6}\lim_{x\to 0} \frac{\sin 4x}{4x}\frac{6x}{\sin 6x} = \frac{4}{6} = \frac{2}{3}.$
because

$$\lim_{x\to 0} \frac{\sin 4x}{4x} = 1 \quad \lim_{x\to 0} \frac{6x}{\sin 6x} = 1$$

(4) 2. Find the limit if it exists. If the limit does not exist then explain why.

$$\lim_{x\to -6} \frac{2x+12}{|x+6|}$$

This limit is of the form 0/0 and so we need a cancellation. Recall the $|x+6| = x+6$ if $x+6 \geq 0$ and $|x+6| = -(x+6)$ if $x+6 < 0$. This suggests that we should look at the limit from the left and right separately.

$$\lim_{x\to -6+} \frac{2x+12}{|x+6|} \left(=\frac{0}{0}\right) = \lim_{x\to -6+} \frac{2x+12}{x+6} = \lim_{x\to -6+} 2 = 2$$

whereas

$$\lim_{x\to -6-} \frac{2x+12}{|x+6|} \left(=\frac{0}{0}\right) = \lim_{x\to -6-} \frac{2x+12}{-(x+6)} = \lim_{x\to -6+} -2 = -2$$

and these two limits are not the same and so the original limit does not exist:

$$\lim_{x\to -6} \frac{2x+12}{|x+6|} \text{ DNE}$$

(4) 3. Let $g(x) = \begin{cases} -x & \text{if } x \leq -1 \\ 1-x^2 & \text{if } -1 < x < 1 \\ x-1 & \text{if } x > 1 \end{cases}$. Evaluate

(a) $\lim_{x \to -1-} g(x) = \lim_{x \to -1-} -x = 1$

(b) $\lim_{x \to -1+} g(x) = \lim_{x \to -1+} 1 - x^2 = 0$

(c) $\lim_{x \to -1} g(x)$ DNE because the limits in Parts (a) and (b) differ.