Quiz 5A, Math 1850 Section 005

10-15-09 Solutions Name ____________________

(3) 1. Find the differential of the function $y = x^2 \sin 2x$

Differentiate. By the product rule.

$$\frac{dy}{dx} = x^2 \frac{d}{dx} \sin 2x + \sin 2x \frac{d}{dx} x^2 = x^2 \cos(2x)2 + \sin(2x)2x = 2x^2 \cos 2x + 2x \sin 2x$$

We used the chain rule to differentiate $\sin 2x$. Therefore the differential is

$$dy = (2x^2 \cos 2x + 2x \sin 2x)dx$$

(3) 2. If $y = x^3 + 2x$ and $dx/dt = 5$, find dy/dt when $x = 2$.

Differentiate in t. Both variables, x and y are functions of time.

$$\frac{dy}{dt} = 3x^2 \frac{dx}{dt} + 2\frac{dx}{dt}$$

Therefore if $dx/dt = 5$ and when $x = 2$ we have $dy/dt = 3(2^2)5 + 2(5) = 70$.

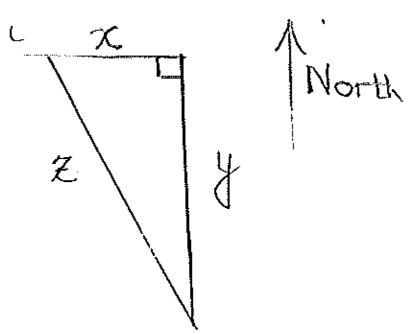
(4) 3. Use a linear approximation (or differentials) to estimate $(8.06)^{2/3}$

A linear approximation is $L(x) = f(a) + f'(a)(x - a)$ Here we want $f(x) = x^{2/3}$, $a = 8$ (where we know something about $f(a) = 8^{2/3} = (2^3)^{2/3} = 2^2 = 4$ and $x = 8.06$. Since $f'(x) = (2/3)x^{-1/3}$ so that $f'(8) = (2/3)8^{-1/3} = (2/3)(1/2) = 1/3$. Therefore

$$L(x) = f(a) + f'(a)(x - a) = 4 + (1/3)(x - 8)$$

A linear approximation of $f(x)$ is $f(8.06) = (8.06)^{2/3} \approx L(8.06) = 4+(1/3)(0.06) = 4.02$.

(10) 4. Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?



Here we know the velocity of the cars and so we make y the distance traveled by the car going south; and x the distance traveled by the car going west. (See picture.) We know that $y' = 60$ and $x' = 25$. We want to know z' where z is the distance between cars. By the Pythagorean theorem $z^2 = x^2 + y^2$. Differentiate in time

$$2zz' = 2xx' + 2yy' \quad \text{or} \quad zz' = xx' + yy'$$

Substitute $y' = 60$ and $x' = 25$. What are x, y and z? Well after 2 hours, the southbound car has gone 120 miles (y=120=60(2)) and the westbound one has gone 50 miles. Therefore $z = \sqrt{x^2 + y^2} = \sqrt{50^2 + 120^2} = 130$. Plugging into the above eequation we have $2(130)z' = 2(50)25 + 2(120)60$ or $z' = (125 + 720)/13 = 65$. The distance between the cars is increasing at 65 mi/h after 2 hours.