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10-29-09

Quiz 7A, Math 1850

Section 005

Name ____________________

(12) 1. Consider the curve $y = \dfrac{x}{x^2 - 9}$.

(a) Find the horizontal and vertical asymptotes.

Solution: There are two vertical asymptotes at $x = \pm 3$ because there is division by 0 at $x = \pm 3$. Because

$$\lim_{x\to\infty} \frac{x}{x^2-9} = \lim_{x\to\infty} \frac{x^2}{x^2} \frac{1/x}{1 - 9/x^2} = \lim_{x\to\infty} \frac{1/x}{1-9/x^2} = 0$$

there is a horizontal asymptote $y = 0$ as $x \to \infty$ and similarly as $x \to -\infty$.

(b) Find the intervals of increase and decrease.

Solution: Differentiate. By the quotient rule

$$\frac{dy}{dx} = \frac{x^2 - 9 - x(2x)}{(x^2-9)^2} = -\frac{9 + x^2}{(x^2-9)^2}$$

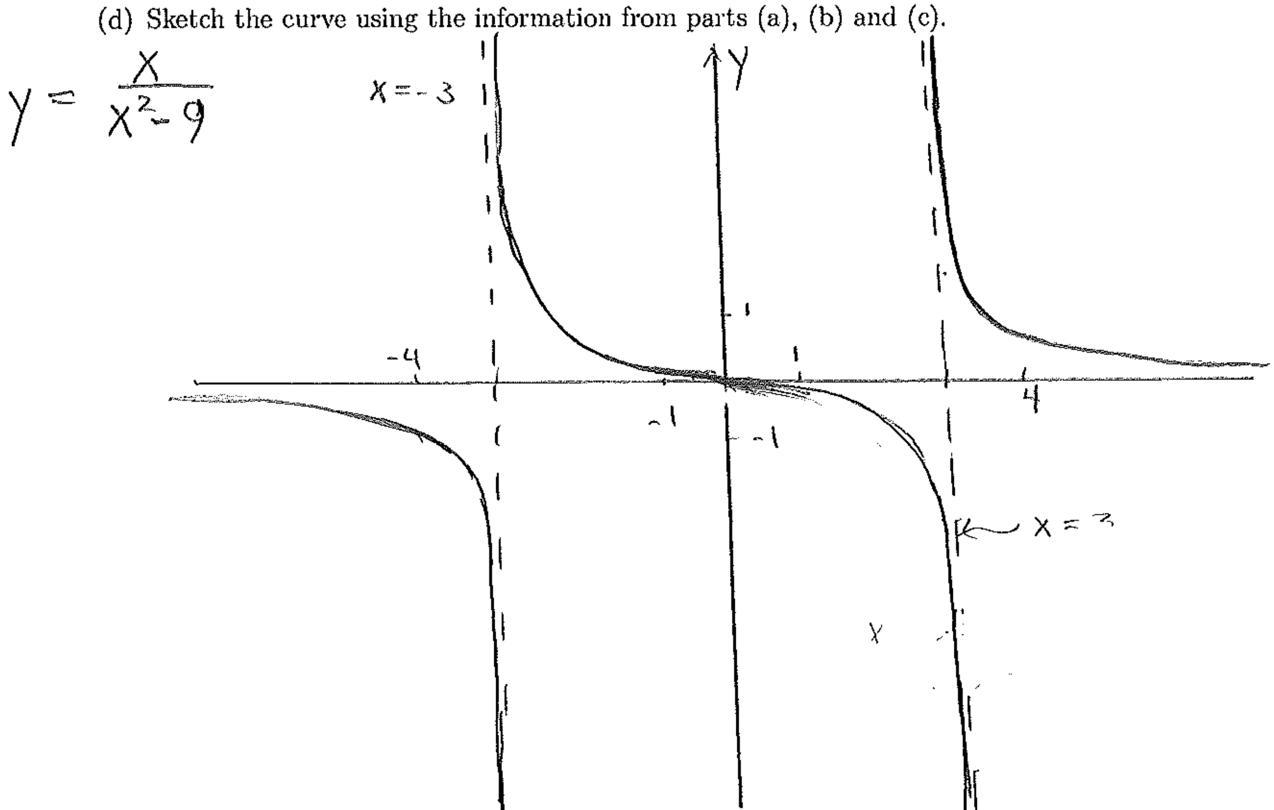
To find the critical points, set the derivative equal to 0: $0 = 9 + x^2$ so that thereare no critical points. The derivative is defined everywhere (not $x = \pm 3$) and so there is no critical point. The intervals of increase and decrease are given by the table below. (Note $f'(x) < 0$ for all x.)

Interval	Evaluate f'	Increasing or Decreasing
$-\infty < x < -3$	$f'(-4) = -25/49$	Decreasing
$-3 < x < 3$	$f'(0) = -1/9$	Decreasing
$3 < x < \infty$	$f'(4) = -25/49$	Decreasing

(c) Find the local maxima and minima.

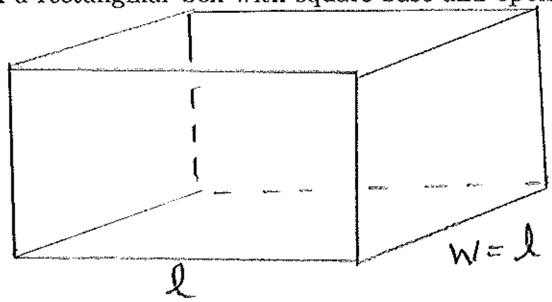
Solution: There is no local maximum or minimum

(d) Sketch the curve using the information from parts (a), (b) and (c).



(8) 2. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Solution: Draw a picture of a rectangular box with square base and open top.



Let the box side lengths be ℓ, w and h. Then $\ell = w$ (square base). We want to maximize $V = \ell^2 h$. We know that the material used is 1200 cm^2: $\ell^2 + 4\ell h = 1200$. Eliminate $h = (1200 - \ell^2)/4\ell = 300\ell^{-1} - \ell/4$ so that $V = \ell^2 h = 300\ell - \ell^3/4$.

We maximize $V = \ell^2 h = 300\ell - \ell^3/4$. We know $\ell \geq 0$ and we know $h \geq 0$ so that $1200 - \ell^2 \geq 0$ or $\ell \leq \sqrt{1200}$ Therefore $0 \leq \ell \leq 20\sqrt{3}$. We can therefore use the closed interval method.

We differentiate $V' = 300 - 3\ell^2/4$ and then set $V' = 0$: $300 - 3\ell^2/4 = 0$ $\ell = 20$. This is the only critical point because V is differentiable on the whole interval.

The endpoints are $\ell = 0$ and $\ell = 20\sqrt{3}$.

Evaluate V at the points found above: $V(0) = 0$; $V(20\sqrt{3}) = 0$ (because $h = 0$) and $V(20) = 6000 - 20^3/4 = 4000$. So the dimensions of the box that maximize volume are $20 \times 20 \times 10$ and the corresponding volume is 4000 cm^2.