Quiz 8B, Math 1850 Section 006

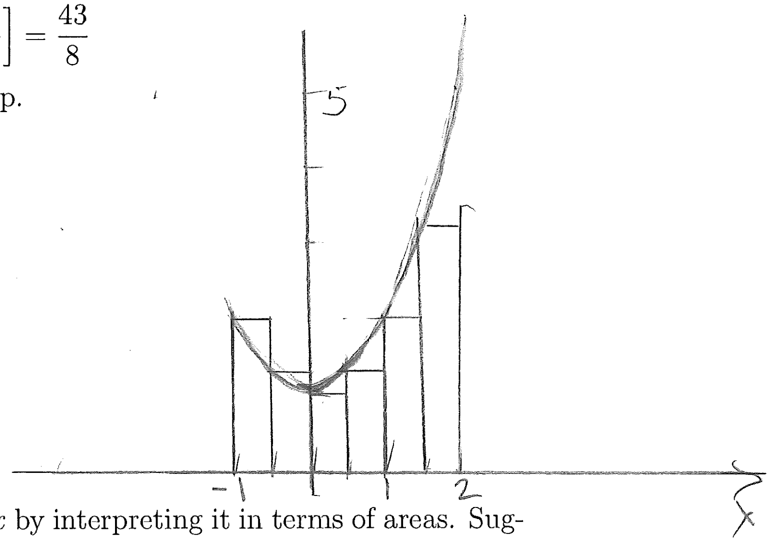
11-19-09 Name ______________________

1. Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using six rectangles and left endpoints.

We want a Riemann sum which corresponds to dividing the interval [-1,2] into 6 equal pieces of length $(2-(-1))/6 = 1/2$. There will be six terms, each one is the area of the box of width 1/2 and height $f(x_i^*)$ where x_i^* is the left endpoint of the ith subinterval, $1 \le i \le 6$. If the area is A then we get

$$\begin{aligned} A &\approx f(-1)\frac{1}{2} + f(-1/2)\frac{1}{2} + f(0)\frac{1}{2} + f(1/2)\frac{1}{2} + f(1)\frac{1}{2} + f(3/2)\frac{1}{2} \\ &= \frac{1}{2}\left[2 + \frac{5}{4} + 1 + \frac{5}{4} + 2 + \frac{13}{4}\right] = \frac{43}{8} \end{aligned}$$

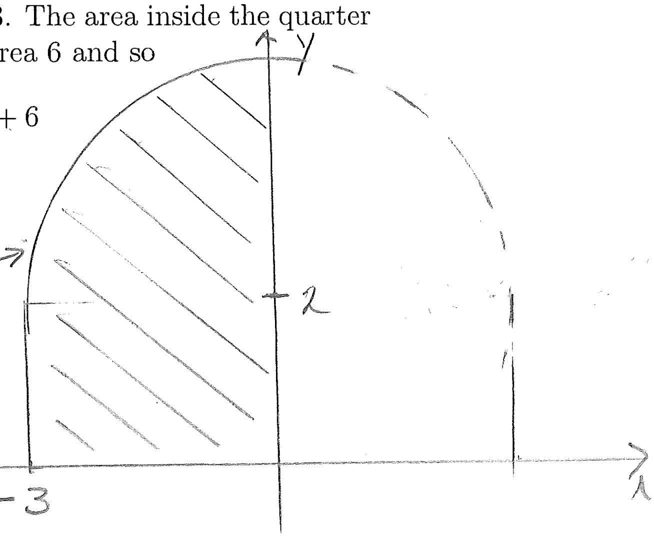
where we factored out 1/2 in the last step.



2. Evaluate the integral $\int_{-3}^{0} (2 + \sqrt{9 - x^2})\, dx$ by interpreting it in terms of areas. Suggestion: Graph the integrand.

The integral represents the area of the region beneath the graph of $y = (2 + \sqrt{9 - x^2})$ but above the portion of the x-axis, $-3 \le x \le 0$. The graph is the curve $y - 2 = \sqrt{9 - x^2}$ or $(y-2)^2 + x^2 = 9$ and this is a circle of radius 3 shifted up 2 units. If we graph it for $-3 \le x \le 0$ we see that the region is a quarter circle of radius 3 sitting on a rectangle of height 2 and width 3. The area inside the quarter circle has area $\pi r^2/4 = 9\pi/4$ and the rectangle has area 6 and so

$$\int_{-3}^{0} (2 + \sqrt{9 - x^2})\, dx = \frac{9\pi}{4} + 6$$



3. Evaluate the integral. (9)

(a) $\int_{-1}^{3} x^5\,dx = \frac{1}{6}x^6|_{-1}^{3} = \frac{1}{6}[3^6 - (-1)^6]\frac{364}{3}$

(b) $\int_{0}^{\pi/4} \sec^2 t\,dt = \tan t|_0^{\pi/4} = \tan(\pi/4) - \tan(0) = 1$

(c)

$$\begin{aligned}\int_1^9 \frac{3x-2}{\sqrt{x}}\,dx &= \int_1^9 3x^{1/2} - 2x^{-1/2}\,dx = 3\frac{2}{3}x^{3/2} - 4x^{1/2}|_1^9 \\ &= 2\cdot 9^{3/2} - 4\cdot 9^{1/2} - [2-4] = 44\end{aligned}$$

4. Find the general indefinite integral.

$$\int (1-t)(2+t^2)\,dt = \int 2 - 2t + t^2 - t^3\,dt = 2t - t^2 + \frac{1}{3}t^3 - \frac{1}{4}t^4 + C.$$

Check by differentiation. $(d/dt)[2t - t^2 + \frac{1}{3}t^3 - \frac{1}{4}t^4] = 2 - 2t + t^2 - t^3.$ (3)