 Final Exam Section 3, 4, 5, 6, 7, 8, 11 (circle one)
12-19-08 Math 1850 Name _____

Justify your work. A nongraphing calculator is permitted but not needed. The exam is 2 hours and 200 points are possible; the value of each question is indicated in the left margin. There are 9 pages, including this one: check that no pages are missing.

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Problem	Points	Score
1*	24	
2*	15	
3	9	
4	12	
5*	9	
6	11	
7	14	
8	14	
9*	12	
10	8	
11	14	
12	12	
13*	18	
14	14	
15	14	
TOTAL	200	

* Common Assessment Questions

1. Differentiate the following functions.

(24)

(a) $f(x) = \dfrac{\sec(x)}{\tan(x) + 2}$.

(b) $h(x) = \sqrt{x + \dfrac{1}{x}}$.

(c) $g(x) = x \csc(2x)$.

(d) $f(x) = \cos\left((1 - x)^2\right)$.

(15) 2. Evaluate the following limits, if they exist.

(a) $\lim_{x\to 3}\left(\frac{x^2+x-12}{x^2-5x+6}\right)$.

(b) $\lim_{t\to 4+}\left(\frac{t}{4-t}\right)$.

(c) $\lim_{x\to\infty}\left(\frac{1-7x+5x^2}{11-x-3x^2}\right)$.

(9) 3. Find dy/dx by implicit differentiation, if $x^2+3xy+y^3=3$.

4. Find all numbers at which the function

(12)

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0, \\ x & \text{if } 0 < x < 1, \\ 2x - 3 & \text{if } x \geq 1. \end{cases}$$

is discontinuous. Explain your answer using, for example, limits or a sketch of the graph.

5. Find an equation for the tangent line to the curve $y = \tan(3x)$ at the point $(\frac{\pi}{12}, 1)$.

(9)

(11) 6. Use the definition of "derivative" to find the derivative of the function

$$f(x) = x^3 - 2x.$$

(14) 7. Evaluate the following integrals:

(a) $\int_{\pi/4}^{\pi/2} \left(\frac{1}{\sqrt{x}} + \sin(2x) \right) dx.$

(b) $\int_0^{\pi/2} (1 + \cos(t))^7 \sin(t)\, dt.$

8. Evaluate the following integrals:

(14)

(a) $\int \frac{x+\sqrt{x}}{x^3}\,dx.$

(b) $\int \frac{z}{(z^2-1)^{\frac{1}{2}}}\,dz.$

9. Find a function $f(x)$ such that $f''(x) = 24x^2+2x+10$, $f(1) = 5$, and $f'(1) = -3$.

(12)

10. Find the area of the region bounded by the y-axis, the line $y = 4$, and the curve $y = \sqrt{x}$.

(8)

11. Find the absolute maximum and minimum of $f(x) = x^3 - 3x + 1$ on the interval $[0, 3]$.

(14)

12. Let $f(x) = x^4 - 4x^3$.

(12)

(a) Find the intervals on which f is increasing and decreasing, and describe its behavior at any critical points.

(b) Find the intervals of concavity and any inflection points.

13. Let $f(x) = \dfrac{x^2}{x^2+9}$.

(18)

(a) Find the vertical and horizontal asymptotes of the graph of f, if any exist.

(b) Find the intervals of increase or decrease.

(c) Find the critical values of f and classify them as local maxima, local minima, or neither.

(d) Use the above information to sketch the graph $y = f(x)$.

14. A car is moving around an elliptical track described by the equation $5x^2 + 2y^2 = 13$. As the car passes through the point (1,-2), its x-coordinate is increasing at 100 km/h. How fast is the y coordinate changing at this instant?

(14)

15. A box with a square base and open top is to have a volume of 64 ft^3. Find the dimensions of the box that minimize the amount of material used in its construction.

(14)