NAME: _____

Math. 1850, sec. 011, Calculus I.

FINAL EXAM

200 POINTS TOTAL

INSTRUCTOR: Ivie Stein Jr.

Thursday, June 24, 2010, 1:30-3:40, UH 4410.

INSTRUCTIONS: Show work. Give details. Show set up. Show how you obtain answer. Simplify answer. No calculators. Turn off cell phones.

1. Evaluate $\lim_{x \to 1} \frac{x^2-1}{x+1}$.

10 pts.

2. Find the derivative by using the definition of a derivative

10 pts.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

where

$$f(x) = x^2.$$

10 pts.

3. Let $f(x) = 2x + 1$ and $g(x) = x^2$.

5 pts. each part

(a) Evaluate $f(g(x))$.

$f(g(x)) =$

(b) Evaluate $g(f(x))$.

$g(f(x)) =$

4. Find an equation for the tangent line to the curve $y = x^2$ at $(2, 4)$.

15 pts.

5. Find $\frac{dy}{dx}$ by implicit differentiation

10 pts. where $x^2 + y^2 = 1$.

6. Consider a sphere of radius r.

2 pts. (a) What is the volume V of the sphere?

$V =$

1 pt. each part (b) What is the surface area S of the sphere?

$S =$

7. Consider a closed rectangular box of height H and base with length L and width W.

2 pts, 1 pt. each part

(a) What is the volume V of the box?

$V =$

(b) What is the total surface area S of the box?

$S =$

8. Use Newton's method with initial approximation $x_0 = 1$ to find x_2 where $f(x) = x^3 + x - 1$.

15 pts,

9. Find $\frac{d}{dx}\left(\int_{t=1}^{t=x} e^{-t^2}\,dt\right)$

10 pts.

10. Find the dimensions r and h of a can for which the surface area

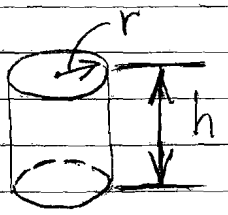
15 pts.

$$A = 2\pi r^2 + 2\pi r h$$

is minimized subject to the volume

$$V = \pi r^2 h = \text{constant}.$$

Refer to the diagram.



11. Use the substitution method to evaluate

10 pts.

$$I = \int \sqrt{x^2-1}\, 2x\, dx.$$

12. Evaluate $I = \int \frac{\ln x}{x} dx.$

10 pts.

13. Evaluate $I = \int_{-1}^{2} x^3 \, dx$.

10 pts.

14. Graph $y = x$ and $y = x^2$ for $-2 \leq x \leq 2$, and find the points of intersection of the two curves. Also find the area bounded by the curves.

15 pts.

20 pts. 15. Let $f(x) = 2x^6 - 5x^4$.

(a) Find $f'(x)$.

2 pts. $f'(x) =$

(b) Find $f''(x)$.

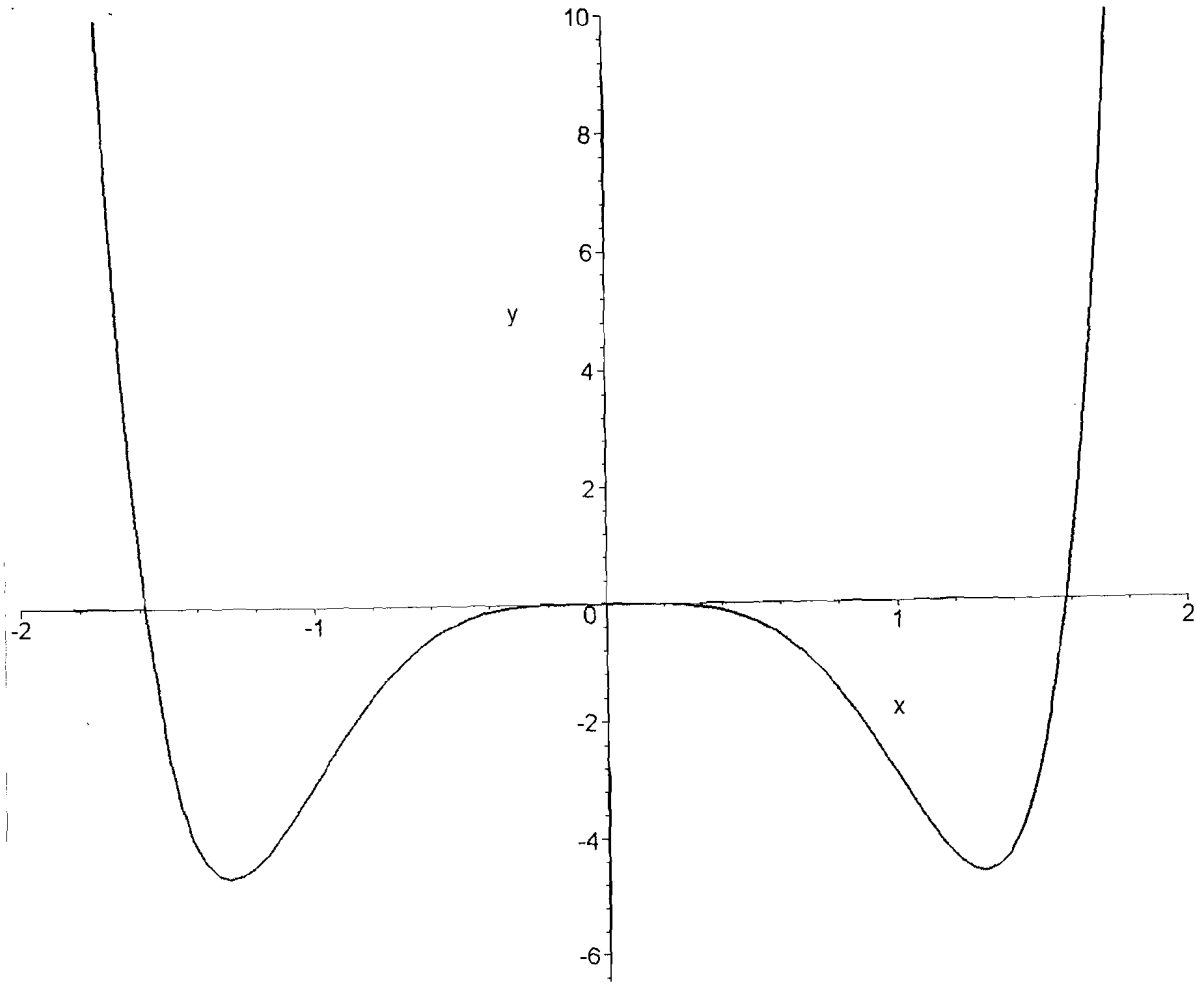
2 pts. $f''(x) =$

(c) Find the critical numbers of f.

2 pts.

(d) On the graph provided to you on the next page, indicate and mark the critical points, points of inflection, local minimums, local maximums, and intervals where f is concave upward and concave downward.

14 pts.


10
8
6
y
4
2
-2
-1
0
1
2
x
-2
-4
-6

16. Evaluate $\frac{d}{dx}\left(\sin^{-1}(3x) + \tan^{-1}(\sqrt{x})\right)$

16 pts.

17. Define $\sinh(x)$.

4 pts.

18. Evaluate $\frac{d}{dx}\left(x^{\cos x}\right)$.

16 pts.